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Point and Interval Estimation.

Point and Interval Estimation for the Mean.

- Point Estimation for the Mean. $\hat{\mu} = \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
- Interval Estimation for the Mean. (Known variance, 100(1 - α) % confidence interval)

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

where $\bar{x}$ = the sample mean.

σ = the population deviation.

$z_{\alpha/2}$ = the critical value of the standard normal distribution.

Point and Interval Estimation.

[CRE Primer Ⅳ 2-11]

Point and Interval Estimation for the Mean.

- Point Estimation for the Mean. $\hat{\mu} = \bar{x} = \dfrac{\sum_{n=1}^{n} x_i}{n}$

- Interval Estimation for the Mean.

  (Unknown mean and variance, 100(1- α) % confidence interval)

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

where $\bar{x}$ = the sample mean.

s = the sample standard deviation.

$t_{\alpha/2}$ = the critical value of the t-distribution.

ϕ = $n-1$ degrees of freedom.

Point and Interval Estimation.

[CRE Primer Ⅳ 2-11]

Point and Interval Estimation for the Variance.

- Point Estimation for the Variance. $\widehat{\sigma^2} = s^2 = \frac{\sum_{n=1}^{n} (x_i - \bar{x})^2}{n-1}$

- Interval Estimation for the Variance. (100(1 - α) % confidence interval)

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},\,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},\,n-1}}$$

where $\chi^2_{\frac{\alpha}{2},n-1}$ = the critical value of the chi-square distribution with $n-1$ degree of freedom.

Point and Interval Estimation.

[CRE Primer Ⅳ 2-11]

Point and Interval Estimation for Proportion.

- Point Estimation for Proportion. $\widehat{P} = p = \frac{d}{n}$

- Interval Estimation for Proportion. (100(1- α) % confidence interval)

$$p - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} < P < p + z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

where p is the proportion of successes.
n = a random sample of size and $np \geq 5$

Point and Interval Estimation.

[CRE Primer Ⅳ 2-11]

MTBF Confidence Intervals : Type Ⅰ Censoring.

- Point Estimation. $\hat{\theta} = \frac{T}{r}$

where n = The number of items originally placed on test.

T = Total testing time or cycles.

r = Number of failures during the test.

- One sided lower confidence limit. $\frac{2T}{\chi^2(\alpha,\ 2r+2)} \leq \theta$

- Two sided confidence interval.

$$\frac{2T}{\chi^2(\frac{\alpha}{2},\ 2r+2)} \leq \theta \leq \frac{2T}{\chi^2(1-\frac{\alpha}{2},\ 2r)}$$

Point and Interval Estimation.

[CRE Primer Ⅳ 2-11]

MTBF Confidence Intervals : TypeⅡ Censoring.

- Point Estimation. $\hat{\theta} = \frac{T}{r}$

where T = Total amount of time on test for all units.
r = Number of failures during the test.

- One sided lower confidence limit. $\frac{2T}{\chi^2(\alpha, 2r)} \leq \theta$

- Two sided confidence interval.

$$\frac{2T}{\chi^2(\frac{\alpha}{2}, 2r)} \leq \theta \leq \frac{2T}{\chi^2(1-\frac{\alpha}{2}, 2r)}$$

Hypothesis Tests.

[CRE Primer Ⅳ 13-35]

Some Concepts.

- The hypothesis being tested is referred to as the null hypothesis, H_0.
- If the null hypothesis is rejected, the alternative hypothesis, H_1, is accepted.
- Type Ⅰ and Type Ⅱ error.

	H_0 : True	H_0 : False
Accept H_0	Correct decision	Type Ⅱ error. (β error)
Reject H_0	Type Ⅰ error. (α error)	Correct decision

Required Sample Size.

[CRE Primer Ⅳ 13-35]

Sample Size needed for Hypothesis Testing.

- The sample size (n) needed for hypothesis testing depends on
  - The desired type I (α) and type Ⅱ (β) risk.
  - The minimum value to be detected between the population means ($\mu - \mu_o$)
  - The variation in the characteristic being measured (s or σ)

- Variable data sample size, only using α.

$$n = \frac{Z^2 \sigma^2}{E^2}$$

where Z = The appropriate Z value.

E = Error desired.

Hypothesis Tests for Means.

[CRE Primer Ⅳ 13-35]

Testing to determine if a population mean is equal to a specific value.

- The null and alternative hypothesis.

H_0 : $\mu = \mu_0$

H_1 : $\mu \neq \mu_0$ or $\mu < \mu_0$ or $\mu > \mu_0$

- The statistic.

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ (If σ is not known)

$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ (If σ is known)

Hypothesis Tests for Means.

[CRE Primer Ⅳ 13-35]

Testing to determine if two population means are equal.

- The null and alternative hypothesis.

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2 \quad \text{or} \quad \mu_1 < \mu_2 \quad \text{or} \quad \mu_1 > \mu_2$$

- Variance are unknown, but consider equal.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where, $s_p = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$ (pooled standard deviation)

$$\phi = n_1 + n_2 - 2$$

Hypothesis Tests for Means.

[CRE Primer Ⅳ 13-35]

Testing to determine if two population means are equal.

- The null and alternative hypothesis.

H_0 : $\mu_1 = \mu_2$

H_1 : $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

- Variance are unknown, but consider unequal.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where $$\phi = \frac{1}{\dfrac{\left(\dfrac{\frac{s_1^2}{n_1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)^2}{n_1 - 1} + \dfrac{\left(\dfrac{\frac{s_2^2}{n_2}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)^2}{n_2 - 1}}$$

Hypothesis Tests for Means.

[CRE Primer Ⅳ 13-35]

Testing to determine if two population means are equal.

- The null and alternative hypothesis.

H_0 : $\mu_1 = \mu_2$

H_1 : $\mu_1 \neq \mu_2$ or $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

- Paired t test. (data is taken in pairs.)

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}}$$

where $\phi = n - 1$

Hypothesis Tests for Variance.

[CRE Primer Ⅳ 13-35]

Testing to determine if a population variance is equal to a specific value.

- The null and alternative hypothesis.

$$H_0 : \sigma = \sigma_0$$

$$H_1 : \sigma \neq \sigma_0 \quad \text{or} \quad \sigma < \sigma_0 \quad \text{or} \quad \sigma > \sigma_0$$

- The test statistic.

$$\chi^2_{\alpha,\ \phi} = \frac{(n-1)s^2}{\sigma_0^2} \qquad \text{where} \quad \phi = n-1$$

$$\chi^2_{\alpha,\ \phi} = \frac{\sum (O-E)^2}{E} \qquad \text{where} \quad \phi = (r-1)(c-1)$$

Hypothesis Tests for Variance.

[CRE Primer Ⅳ 13-35]

Testing to determine if there is a difference in the variance of two populations.

- The null and alternative hypothesis.

H_0 : $\sigma_1 = \sigma_2$

H_1 : $\sigma_1 \neq \sigma_2$ or $\sigma_1 < \sigma_2$ or $\sigma_1 > \sigma_2$

- The test statistic.

$$F_{\alpha,\ \phi_1,\ \phi_2} = \frac{s_1^2}{s_2^2}$$ where $\phi_1 = n_1 - 1, \quad \phi_2 = n_2 - 1$

Goodness of Fit Tests.

[CRE Primer Ⅳ 13-35]

Introduction to GOF Tests.

- GOP tests are part of a class of procedures that are structured in cells. In each cell there is an observed frequency (F_o). One either knows from the nature of the problem the expected or theoretical frequency (F_e) or one can calculate it.

$$\chi^2 = \frac{\sum (F_o - F_e)^2}{F_e}$$

Degree of freedom (DF)

Normal Distribution : 3

Poisson Distribution : 2

Binomial Distribution : 2

Uniform Distribution : 1

Hypothesis Tests for Variance.

[CRE Primer Ⅳ 38]

Parametric vs. Nonparametric Tests.

- Parametric test implies that a distribution is assumed for the population. Often, an assumption is made when performing a hypothesis test that the data is a sample from a certain distribution.

- An advantage of a parametric test is that if the assumptions hold, the power, or the probability of rejecting H_0 when it is false, is higher than is power of a corresponding nonparametric test with equal sample size.

- Nonparametric techniques of hypothesis testing are applicable for many quality engineering problems and projects. The nonparametric tests are often called "distribution-free" since they make no assumption regarding the population distribution.

- Three powerful nonparametric techniques : Kendall Coefficient of Concordance, Spearman Rank Correlation Coefficient (R_s), and Kruskal-Wallis One way ANOVA.

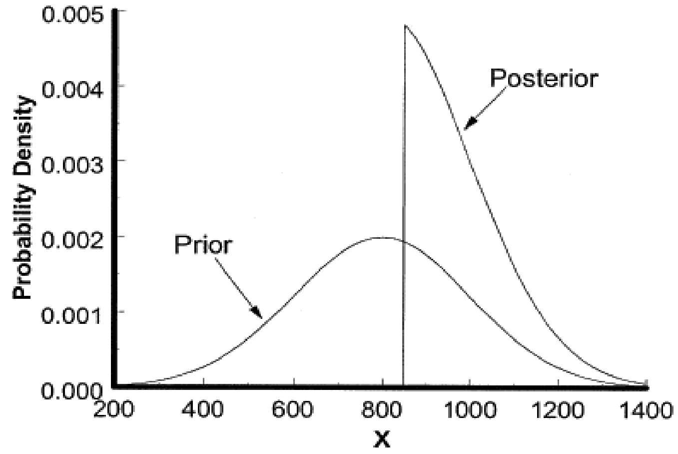
Bayesian Technique.

[CRE Primer Ⅳ 46-49]

Prior and Posterior Probability.

- Bayes' theorem is based on a prior distribution and a posterior distribution.

- For example, an item has a normally distributed time to fail distribution with a mean of 800 hours, and a standard deviation of 200. What is the probability of a item surviving for 900 hours ? Given an item has survived for 850 hours, what is the probability of surviving for 900 hours ? The first question is answered using the prior distribution, and the second question is answered using the posterior distribution.



Bayesian Technique.

[CRE Primer Ⅳ 46-49]

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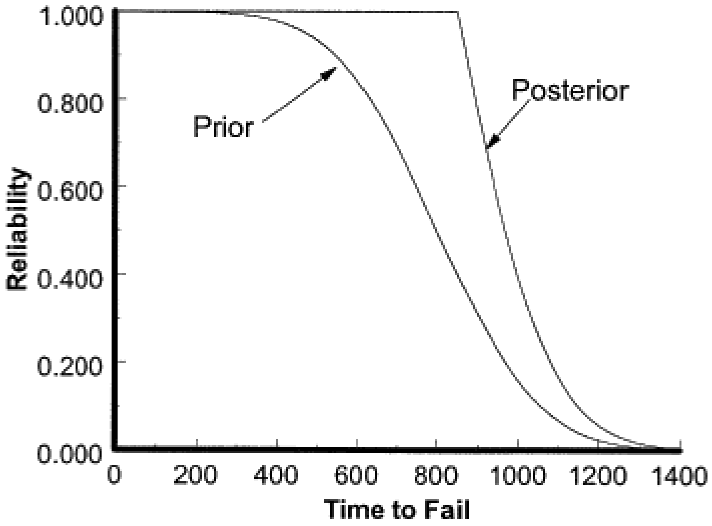
Posterior Reliability Function.



- The posterior probability density function : $f_{posterior}(x) = \dfrac{f_{prior}(x)}{R(x_0)}$

- The posterior reliability function : $R_{posterior}(x) = \dfrac{R_{prior}(x)}{R(x_0)}$

Bayesian Technique.

[CRE Primer Ⅳ 46-49]

Posterior Reliability Function.

- Continuing with the example, the probability of survival for 850 hours is 0.4013. The probability of survival for 900 hours given survival for 850 hours is equal to the reliability at 850 hours divided by the reliability at 900 hours.

$$R(900|850) = \frac{R(900)}{R(850)} = \frac{0.3085}{0.4013} = 0.7688$$

- In case of exponential distribution.

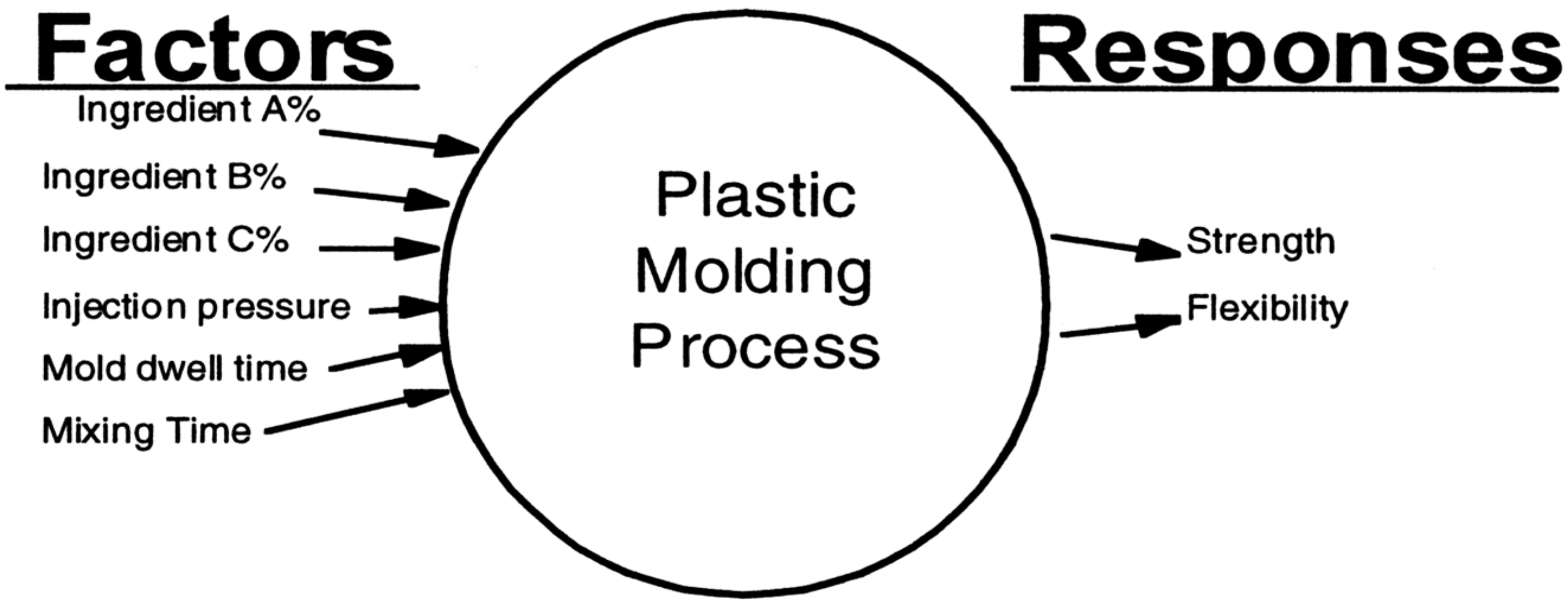
$$R(t+\triangle t \mid t) = \frac{R(t+\triangle t)}{R(t)} = \frac{e^{-\lambda(t+\triangle t)}}{e^{-\lambda t}} = e^{-\lambda \triangle t} = R(\triangle t)$$

- Note that the probability of survival in the interval from 0 to 100 hours is equal to the probability of survival in the interval from 100 t0 200 hours given survival for 100 hours. This is known as the "lack of memory" property, which is unique to the exponential and Poisson distribution.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Introduction to DOE.



Design of Experiment.

[CRE Primer Ⅳ 50-88]

Experimental Objectives.

- Comparative objectives.
- Screening objectives.
- Response surface (method) objective.
- Optimizing responses when factors are proportions of a mixture objectives.
- Optimal fitting of a regression model objective.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Introduction to DOE.

- The traditional approach of changing one variable at a time has shortcomings :
  - Too many experiments are necessary to study the effects of all the input factors.
  - The optimum combination of all the variables may never be revealed.
  - The interaction (the behavior of one factor may be dependent on the level of another factor) between factors cannot be determined.

- Statistical Design of Experiments (SDE).
  - SDE overcomes the above problems by careful planning.
  - SDE is a methodology of varying many input factors simultaneously in a carefully planned manner, such that their individual and combined effects on the output can be identified.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Randomized Block Plans.

- The required number of tests may be to large to be carried out under similar conditions. In such cases, one may be able to divide the experiment into blocks, or planned homogeneous groups.

- When each group in the experiment contains exactly one measurement on every treatment, the experimental plan is called a randomized block plan.

- For example, an experimental scheme may take several days to complete. If one expects some biased differences among days, one might plan to measure each item on each day, or to conduct one test per day on each item. A day would them represent a block.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Randomized Block Plans.

- Only treatment A, C, and D are run on the first day. B, C, and D on the second day, etc. In the whole experiment, note that each pair of treatments, such as BC occurs twice together. The order in which the three treatments are run on a given day follows a randomized sequence.

	Treatment			
Block (Days)	A	B	C	D
1	-5	omitted	-18	-10
2	omitted	-27	-14	-5
3	-4	-14	-23	omitted
4	-1	-22	omitted	-12

Latin Square Designs.

- It is often useful when it is desirable to allow for two sources of non-homogeneity in the conditions affecting test results.

- Restriction.
  - The number of rows, columns and treatments must be the same.
  - There should be no interactions between row and column factors, since these cannot be measured.

- A Latin square design is essentially a fractional experiment which requires less experimentation to determine the main treatments results.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Latin Square Designs.

Car	Carburetor Type				
	I	II	III	IV	V
1	A	B	C	D	E
2	B	C	D	E	A
3	C	D	E	A	B
4	D	E	A	B	C
5	E	A	B	C	D

	I	II	III
A	1	2	3
B	2	3	1
C	3	2	1

	I	II	III
A	1	3	2
B	2	1	3
C	3	2	1

	I	II	III
A	2	1	3
B	3	2	1
C	1	3	2

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Graeco-Latin Square Designs.

- Graeco-Latin square designs are sometimes useful to eliminate more than two sources of variability in an experiment. A Gaeco-Latin square is an extension of the Latin square design but one extra blocking variable is used.

	I	II	III
A	11	23	32
B	22	31	13
C	33	12	21

Design of Experiment.

[CRE Primer Ⅳ 50-88]

Graeco-Latin Square Designs.

- 4 × 4 Graeco-Latin square : * Driver : A, B, C, D ** Days : α β γ δ

Car	Carburetor Type			
	I	II	III	IV
1	A α	B β	C γ	D δ
2	B δ	A γ	D β	C α
3	C β	D α	A δ	B γ
4	D γ	C δ	B α	A β

[CRE Primer Ⅳ 50-88]

Hyper-Graeco-Latin Square Designs.

- A Hyper-Graeco-Latin square design permits the study of treatments with that three blocking variables.

- 4 × 4 Hyper- Graeco-Latin square :

  * Driver : A, B, C, D ** Days : α β γ δ

  *** Tires : M, N, O, P **** Speeds : Φ, X, Ψ, Ω

Car	Carburetor Type			
	I	II	III	IV
1	A α M Φ	B β N X	C γ O Ψ	D δ P Ω
2	B δ N Ω	A γ M Ψ	D β P X	C α O Φ
3	C β O X	D α P Φ	A δ M Ω	B γ N Ψ
4	D γ P Ψ	C δ O Ω	B α N Φ	A β M X

Design of Experiment.

[CRE Primer Ⅳ 50-88]

A Full and Fraction Factorial Design.

- A full factorial.
  - An experimental design which contains all levels of all factors.
  - No possible treatments are omitted.

- A fractional factorial.
  - A balanced experimental design which contains fewer than all combinations of all levels of all levels of all factors.

$$K^n$$

where K = No. of levels

n = No. of factors.

Design of Experiment.

[CRE Primer Ⅳ 50-88]

A Full and Fraction Factorial Design.

Full Factorial

Exp.	T	P	C
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

Half Fractional Factorial

Exp.	T	P	C
1	−	−	−
4	+	+	−
6	+	−	+
7	−	+	+

Analysis of Variance.

[CRE Primer Ⅳ 89-111]

Introduction.

- There are underlying assumptions in ANOVA.
  - The variation within each factor is the same.
  - the distribution are normal and the error is independent.

- With ANOVA, the variation in response measurement are partitioned into components that reflect the effects of one or more independent variables.

- When an experimental variables is highly related to the response, its part of the total sum of the squares will be highly inflated. This condition is confirmed by comparing the variable sum of squares with that of the random error using an F test.

Analysis of Variance.

[CRE Primer Ⅳ 89-111]

ANOVA Step.

Step 1. Calculate the sum of square.

- Total SS = $SST+SSE$
- Total SS = $\sum$ (each observation - $\overline{X}$)^2

  = $\sum$ (each observation)^2 - CM(correction for the mean)

- SST = The sum of squares between treatments = $\frac{n_1 n_2}{n_1+n_2}(\overline{X}_1-\overline{X}_2)^2$
- SSE = The sum of squares with treatments.

Step 2. Calculate the mean variation.

- MST = Mean square of treatments = $\frac{SST}{\text{treatments}-1}$
- MSE = $\frac{SSE}{n_1+n_2-2}$

Analysis of Variance.

[CRE Primer Ⅳ 89-111]

ANOVA Step.

Step 3. The test statistic for the null hypothesis.

- $H_0 : \mu_1 = \mu_2$
- $F_{cal} = \frac{MST}{MSE}$

Factor	Sum of Square	Degree of freedom	Mean Variance	F$_{cal}$	F$_{critial}$
A	$\sum_{i=1}^{l} \frac{T_{i.}^2}{m} - CT$	$\phi_A = l-1$	$\frac{S_A}{\phi_A}$	$\frac{V_A}{V_E} > 1$	$F(\phi_A, \phi_E, a)$
E	$S_T - S_A$	$\phi_E = l(m-1)$	$\frac{S_E}{\phi_E}$		
T	$\sum_{i=1}^{l}\sum_{j=1}^{m} x_{ij}^2 - CT$	$\phi_T = l \cdot m - 1$			

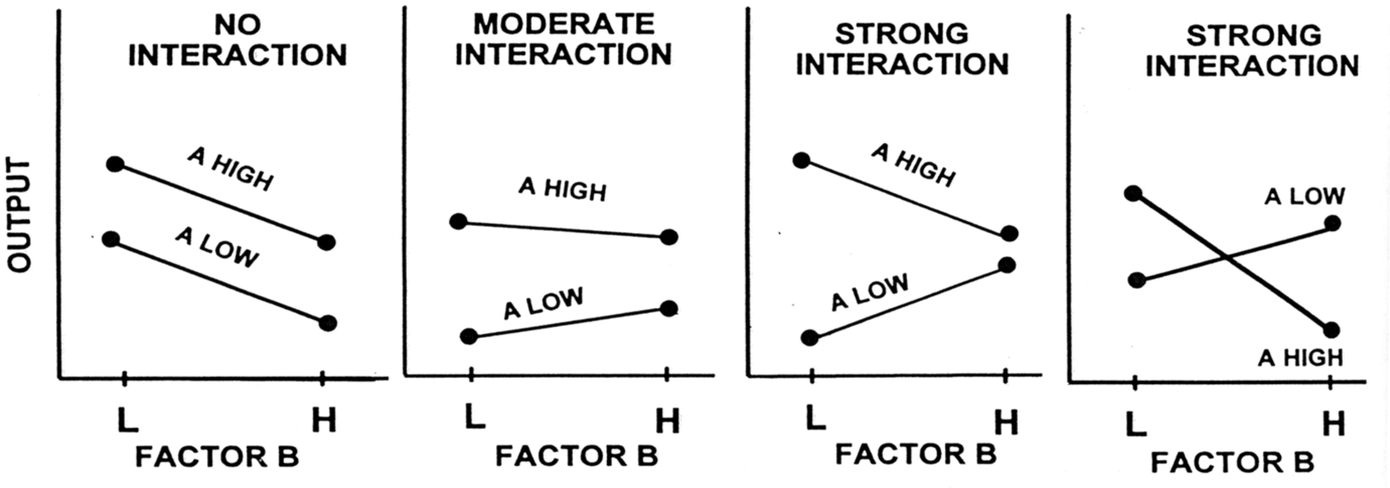
Analysis of Variance.

[CRE Primer Ⅳ 89-111]

Interaction.

- An interaction occurs when the effect of one input factor on the output depends upon the level of another input factor.

- Interaction diagram.



[CRE Primer Ⅳ 89-111]

A × B Factorial Experiment.

- In A×B factorial experiment, the sum of squares for treatments can be partitioned into the sum of squares for factors A and B, and the sum of squares for the A/B interaction.
- The total sum of squares can be partitioned into

$$\text{Total } SS = SS(A) + SS(B) + SS(AB) + SSE$$

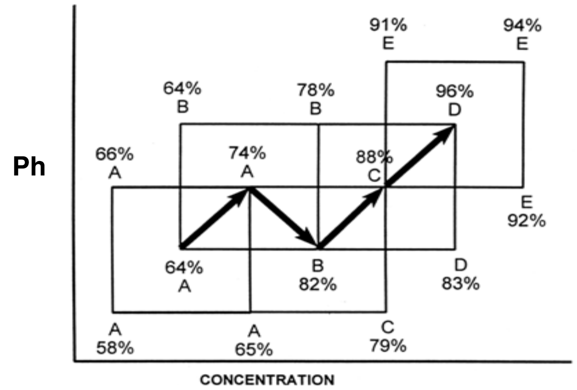
Factor	Sum of Square	Degree of freedom	Mean Variance	F_{cal}
A	$SS(A)$	$(a-1)$	$MS(A) = SS(A)/(a-1)$	$MS(A) / MSE$
B	$SS(B)$	$(b-1)$	$MS(B) = SS(B)/(b-1)$	$MS(B) / MSE$
$A \times B$	$SS(AB)$	$(a-1)(b-1)$	$MS(AB) = SS(AB)/(a-1)(b-1)$	$MS(AB) / MSE$
E	SSE	$(n-ab)$	$MSE = SSE/(n-ab)$	
T	$Total\ SS$	$(n-1)$		

Evolutionary Operation.

[CRE Primer Ⅳ 89-111]

EVOP.

- Many experiment designs emphasize a bold strategy for improvement. In contrast EVOP emphasizes a conservative experimental strategy of continuous process improvement.
- Tests are carried out in phase *A* until a response pattern is established. Then phase *B* is centered on the best conditions from phase *A*. This procedure is repeated until the best result is determined.



Regression.

[CRE Primer Ⅳ 89-111]

Introduction.

- A method of fitting an equation to a data set. Simple regression involves one independent variable and one dependent variable. Least squares is the most common method of regression. With least squares regression a straight line is fit to the data by minimizing the sum of the squared distance of each point from the line.

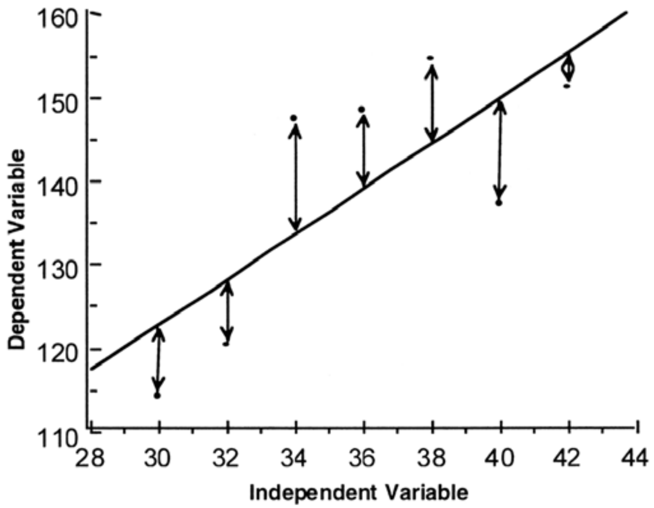


Figure 4. Least squares regression.

Regression.

[CRE Primer Ⅳ 89-111]

Introduction.

- The coefficient of determination : r^2
  - A measure of the amount of variability in the data explained by the regression model. If r^2 = 0.91, then 91% of the variability in the model is explained by the regression model.

- The correlation coefficient : r
  - A measure of correlation between the independent variable(s) ad the dependent variable.
  - Perfect positive correlation : $r = 1$
  - Perfect negative correlation : $r = -1$

Regression.

[CRE Primer Ⅳ 89-111]

A Simple Linear Model.

- $y_i = a + bx_i + e_i$

where y_i is the dependent variable.

x_i is the independent variable, and

e_i is the residual ; the error in the fit of the model.

- Using the method of least squares, the coefficients are estimated fro the following expression.

- $$b = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{S_{xy}}{S_{xx}}$$

- $$a = \frac{\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i}{n} = \bar{y} - b\bar{x}$$

[CRE Primer Ⅳ 89-111]

Correlation Coefficient.

- An indicator of the strength of th linear relationship between two variables y and x is called the Pearson product-moment coefficient of correlation, where

$$r=\frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$