

Certified Reliability Engineer.



中国最专业、最有影响力的可靠性行业网站

# Ch.5 Reliability System Modeling.

<https://www.kekaoxing.com>

Industrial Engineering & Management System Research Center.

# Reliability Data.

---

[CRE Primer VI 2-6]

## Sources of Reliability Data.

- Successful reliability programs must include the capability to model and predict the reliability of both components and systems. These predictions are usually based on data from actual experiences.
- The general concept is that component or assembly life characteristics remain constant so that they may be used to predict future behavior of the general population.
- Reliability data from the two sources.
  - Internal Sources : Internal Testing, Field Data.
  - External Sources : Industry Data, Public Sources.

# Reliability Data.

---

[CRE Primer VI 2-6]

## Internal Testing.

- The important features which must be captured in this data are the conditions, procedures, and models (or versions) of the parts or assemblies that were tested. The types of internal testing for reliability data.
  - Research tests.
  - Prototype tests.
  - Environmental tests.
  - Development and reliability growth tests.
  - Qualification tests.
  - Tests on purchased items.
  - Production assessment and production acceptance tests.
  - Tests of failed or malfunctioning items.

# Reliability Data.

---

[CRE Primer VI 2-6]

## Necessary Information from Internal Testing.

1. The component or assembly test information.
2. Environment.
3. Length of test.
4. Description of failure.



# Reliability Data.

---

[CRE Primer VI 2-6]

## Necessary Information from Internal Testing.

### 1. The component or assembly test information.

- Describing what was actually tested.
- Including any information necessary to specifically detail the version or revision of the item, the source of the item and the type of device.

### 2. Environment.

- Useful reliability tests are almost always run at environments and conditions that are equal to or greater than the expected use conditions.
- In some applications, accelerating testing is used. Accelerated testing is generally used to reveal design or material flaws. In some cases, accelerated life testing is used to predict reliability. However, the relationship between accelerated test data and normal environments is frequently difficult to accurately predict.

# Reliability Data.

[CRE Primer VI 2-6]

## Necessary Information from Internal Testing.

### 3. Length of test.

- The total length of time of all successful tests, as well as the times to failure of any failed components or units.

### 4. Description of failure.

- The failure type is very important to the reliability data set. The critical parameters for reliability prediction are the causes of the failure and whether the failure causes are inherent in the design.
- Some types of failures.  
Design defects, Manufacturing defects, Improper defects, Secondary failure (caused by a preceding failure), Intermittent or transient failures, Wear-out failure, Failures of unknown origin.

# Reliability Data.

---

[CRE Primer VI 2-6]

## Field Data.

- Failure from units in the field provide the most valuable source of data to the design company. Field failures are identified by
  - Warranty returns.
  - Customer complaints.
  - Field representative information.
  - Distributor/dealer information.
- Field failures must be analyzed for the same types of failures discussed in internal testing. The stress and imposed conditions must be determined.
- Another source of reliability data available is from the supplies of sub-components of the system. Every major sub-component should be purchased with both reliability requirements and accurate reliability information provided.

# Reliability Data.

---

[CRE Primer VI 2-6]

## Industry Data.

- Some industries have organizations that share reliability data. Data sharing has generally been useful for electronic components than mechanical components.
- One example of industry data is through the Institute of Electrical and Electronics Engineers (IEEE), which maintains a large data bank of information on electronic hardware.



# Reliability Data.

---

[CRE Primer VI 2-6]

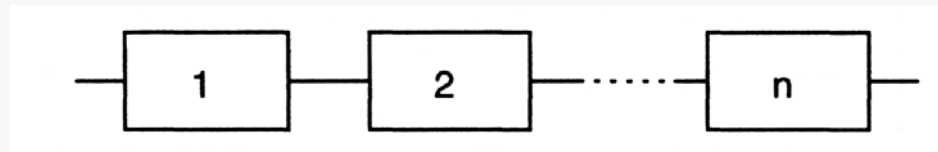
## Public Data.

- The US government has been a leader in obtaining and sharing reliability information. The most extensive is the Government-Industry Data Exchange Program (GIDEP). This program is the cooperative venture between government and industry to share reliability data.
- The GIDEP program maintains four data interchanges :
  - Engineering data interchange.
  - Reliability-maintainability data interchange.
  - Metrology data interchange.
  - Failure experience data interchange.

# System Reliability.

[CRE Primer VI 7-20]

## Series Model.



- For a series system to operate successfully, all components must operate successfully.
- The reliability of a series :  $R_s = \prod_{i=1}^n R_i$
- The hazard function :  $\lambda_s = \sum_{i=1}^n \lambda_i$
- The system mean time to failure :  $MTTF_s = \frac{1}{\sum_{i=1}^n \lambda_i}$

# System Reliability.

[CRE Primer VI 7-20]

## Active Redundancy.

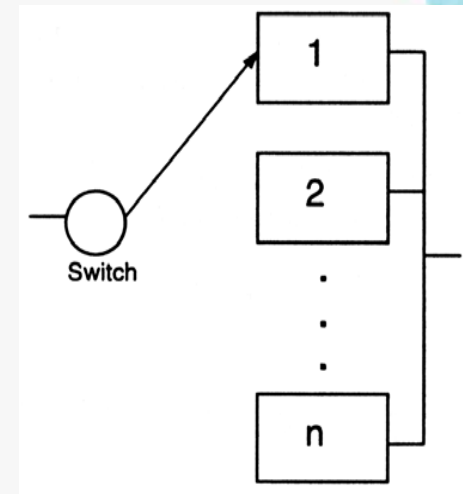
- Reliability of a parallel :  $R_s = 1 - \prod_{i=1}^n (1 - R_i)$
- The system mean time to failure :  $MTBF_s = \sum_{i=1}^n \frac{1}{i\lambda}$

## Stand by Redundancy.

- Reliability of stand by redundancy =  $\sum_{i=0}^n e^{-\lambda t} \left\{ \frac{(\lambda t)^i}{i!} \right\}$
- For three components in spares =

$$R(t) = e^{-\lambda t} \left[ 1 + \lambda t + \frac{(\lambda t)^2}{2!} + \frac{(\lambda t)^3}{3!} \right]$$

- A configuration consisting of two operating components, backed by two spares.  $R(t) = e^{-2\lambda t} \left[ 1 + 2\lambda t + \frac{(2\lambda t)^2}{2!} \right]$



# System Reliability.

[CRE Primer VI 7-20]

## r out of n Model.

- A system consisting of  $n$  components in which  $r$  of the  $n$  components must function for the system to function.
- The reliability of an  $r$ -out-of- $n$  system.

$$R_s = \sum_{x=r}^n \binom{n}{x} R^x (1-R)^{n-x}$$

(where  $R$  is the reliability of all component and is equal)

- The system mean time to fail (where the component failure rates are constant)

$$MTBF_s = \sum_{i=r}^n \frac{1}{i\lambda}$$

( $\lambda$  is the failure rate of each of the components)

# System Reliability.

[CRE Primer VI 7-20]

## Other Reliability Redundancy System.

- **Standby Redundancy, Unequal Failure Rates, Perfect Switching.**

$$R(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

where  $\lambda_1$  = The failure rate for the primary unit.

$\lambda_2$  = The failure rate for the backup unit.

$t$  = Total time.

- **Standby Redundancy, Equal Failure Rates, Imperfect Switching.**

$$R(t) = e^{-\lambda t} [1 + R_{(SW)} \times \lambda t]$$

where  $R_{(SW)}$  = Reliability of the switch.



# System Reliability.

[CRE Primer VI 7-20]

## Other Reliability Redundancy System.

- **Standby Redundancy, Unequal Failure Rates, Imperfect Switching.**

$$R(t) = e^{-\lambda_1 t} + R_{(SW)} \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

- **Shared Load parallel systems.**

An active parallel system where both items are active during operation. On failure of one of the items, the surviving item is left to carry the entire load and as a result the failure rate is increased from the shared load failure rate.

$$R(t) = e^{-2\lambda_1 t} + \frac{2\lambda_1}{(2\lambda_1 - \lambda_2)} (e^{-\lambda_2 t} - e^{-2\lambda_1 t})$$

where  $\lambda_1$  = Failure rate for each item when both are functioning.

$\lambda_2$  = Failure rate of surviving item when one item has failed.

# System Reliability.

---

[CRE Primer VI 21-33]

## Reliability Calculation for Complex System.

- The reliability block diagram.
- Bayes' Theorem application.
- Boolean truth table method.
- Tie set and cut set method.
- Fault-tree analysis.

# System Reliability.

[CRE Primer VI 21-33]

## Bayes' Theorem Application.

- A reliability diagram cannot be reduced easily to a series or parallel system, the application of Bayes' theorem may be employed. In the application of Bayes's theorem, the identification of a keynote component is necessary.
- Characteristics of a keynote component.
  1. A component that enhance reliability of the system by its addition, but still lets the system operate if it fails.
  2. A form of partial redundancy.
- The probability of system failure :

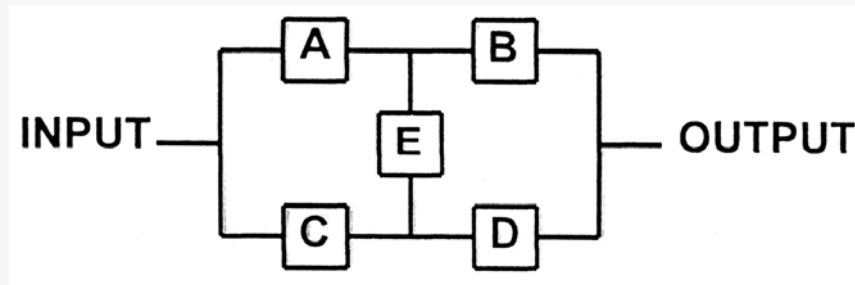
$$P_{(F)} = P(\text{system failure if } E \text{ is good}) \times P(E \text{ is good}) \\ + P(\text{system failure if } E \text{ is bad}) \times P(E \text{ is bad}) \\ \text{(where } E \text{ is a keynote component)}$$

# System Reliability.

[CRE Primer VI 21-33]

## Bayes' Theorem Application : Example.

- The reliability modeling of the complex system using Bayes' Theorem.



- The reliability for component **A** for 10 hr = 0.99079.
- The reliability for component **B** for 10 hr = 0.95123.
- The reliability for component **C** for 10 hr = 0.94176.
- The reliability for component **D** for 10 hr = 0.93239.
- The reliability for component **E** for 10 hr = 0.97045.

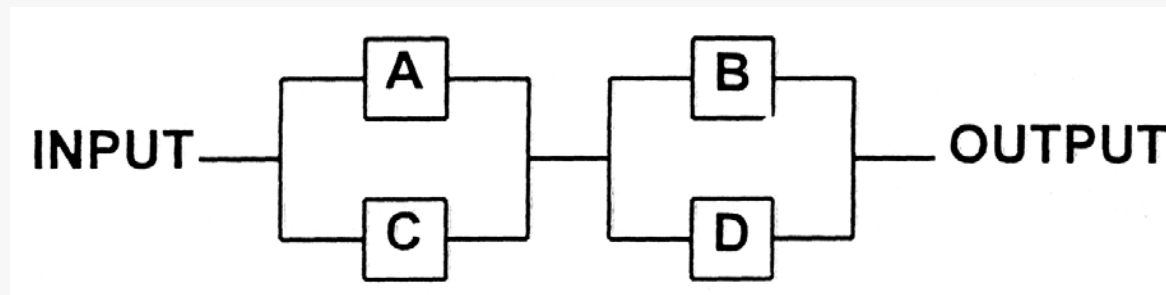
# System Reliability.

[CRE Primer VI 21-33]

## Bayes' Theorem Application : Solution.

Step1. Calculate the probability when a keynote component, **E**, is good.

$$\begin{aligned} & - P(\text{Failure if } E \text{ is good}) \times P(E \text{ is good}) \\ & = (0.00557) \times (0.97045) = 0.00541 \end{aligned}$$





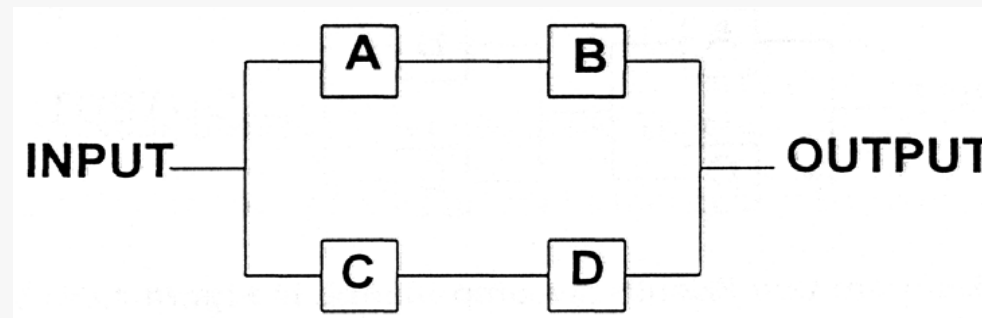
# System Reliability.

[CRE Primer VI 21-33]

## Bayes' Theorem Application : Solution.

Step2. Calculate the probability when a keynote component, **E**, is bad.

$$\begin{aligned} & - P(\text{Failure if } \mathbf{E} \text{ is bad}) \times P(\mathbf{E} \text{ is bad}) \\ & = (0.01049) \times (0.02955) = 0.00031 \end{aligned}$$



Step3. Calculate the total system unreliability : The sum of Steps 1 and 2.

$$- \text{The total system unreliability} = 0.00541 + 0.00031 = 0.00572$$

Step4. Calculate the system reliability

$$- \text{The system reliability} = 1 - \text{the total system unreliability} = 1 - 0.00572 = 0.99428$$

# Reliability Prediction.

[CRE Primer VI 40-41]

## Part Count and Part Stress Method.

- MIL-HDBK-217, Reliability Prediction of Electronic Equipment, presents two methods to estimate the reliability of electronic equipment during design. These methods are known as the "Parts Count" and "Part Stress" methods.

- **Part Count Method.**

The parts counts method simply sums the estimated failure rates of the components in an assembly. This technique assumes a series reliability model. The equipment failure rate can be expressed as :

$$\lambda_{equip} = \lambda_1 + \lambda_2 + \dots + \lambda_n$$

where  $\lambda_{equip}$  = Total equipment failure rate.

$\lambda_n$  = Failure rate of the  $n^{th}$  component.

# Reliability Prediction.

[CRE Primer VI 40-41]

## Part Count and Part Stress Method.

- **Part Stress Method.**

The Part Stress Analysis is a similar calculation for estimated individual part failure rates. This method uses the expression :

$$\lambda_p = \lambda_b \times \Pi_Q \times \Pi_e$$

where  $\lambda_p$  = Estimate of individual part failure rate.

$\lambda_b$  = Base failure rate.

$\Pi_Q$  = Quality factor from above.

$\Pi_e$  = Environmental stress factor.

# Reliability Life Stage Data.

---

[CRE Primer VI 42-43]

## Concept and Planning Stage.

- This is the starting stage, in which marketing, design and management outline the design requirements, including reliability.
- Reliability information sources.
  - Existing product information.
  - Part counts.
  - Part stress.
  - Industry data.

# Reliability Life Stage Data.

---

[CRE Primer VI 42-43]

## Design and Development Stage.

- The design and development stage frequently involves component and subsystem verification testing.
  - Verification testing.
  - Vendor information.
  - Stress analysis.

## Production Stage.

- During production more test results are available, and therefore the reliability information is increasingly more accurate.
  - Validation test results.
  - Final qualification test results.
  - Quality control information on component quality level.



# Reliability Life Stage Data.

---

[CRE Primer VI 42-43]

## Deployment Stage.

- After deployment, the results from customer use are available.
  - Field item returns.
  - Warranty informations.
  - Customer complaints.
  - Field representative data.
  - Repair facility information.

# Reliability Prediction Uncertainty.

[CRE Primer VI 43-47]

## Reliability Prediction at Useful Life.

- During useful life the reliability of a system or component is characterized by the exponential distribution.
- The exponential distribution.
  - Failure density function :  $f(t) = e^{-\lambda t}$
  - Mean of the exponential distribution :  $\mu = \frac{1}{\lambda}$
  - Variance of the exponential distribution :  $\sigma^2 = \left(\frac{1}{\lambda}\right)^2$
- The standard deviation of the exponential distribution is equal to  $1/\lambda$ , or the MTBF. Conservative predictions of reliability use this expectation band.

<https://www.kekaoxing.com>

# Reliability Prediction Uncertainty.

[CRE Primer VI 43-47]

## Reliability Prediction at Wear out Period ( $\sigma$ is known).

- **Reliability Prediction Procedure.**

1. Determine the required reliability level and the confidence level.
2. Determine the value of the standard deviation.
3. Obtain the sample size  $n$ .
4. Estimate the value for the mean life  $\bar{X}$ .
5. Calculate the standard error of the mean  $\frac{\sigma}{\sqrt{n}}$ .
6. Calculate the lower confidence of the mean life.  $X_L = \bar{X} - Z_\alpha \frac{\sigma}{\sqrt{n}}$
7. Calculate the lower limit for reliability.  $X_{(1-\alpha)} = X_L - Z_\alpha \sigma$

# Reliability Prediction Uncertainty.

[CRE Primer VI 43-47]

## Reliability Prediction at Wear out Period ( $\sigma$ is unknown).

- Reliability Prediction Procedure.

- Upper and lower tolerance limit :  $X_{UL} = \bar{X} \pm k_2s$

- Upper or lower tolerance limit :  $X_L = \bar{X} - k_1s, \quad X_U = \bar{X} + k_1s$

where  $\bar{X}$  = sample mean.

$k_1$  = one-sided tolerance factor.

$k_2$  = two-sided tolerance factor.

# Reliability Apportionment.

[CRE Primer VI 50-53]

## Equal Apportionment Technique.

- This technique is a straightforward method that assigns equal reliability requirements for all subsystems based on the system requirements. While being easy to compute, it lacks the sophistication to discriminate between actual subsystem reliabilities.

$$R_{\text{sys}} = (R_1)(R_2) \cdots (R_n)$$

where  $R_1, R_2, \cdots, R_n$  are individual subsystems.

$$\therefore R_i = (R_{\text{sys}})^{\frac{1}{n}}, \quad i = 1, 2, \cdots, n$$

# Reliability Apportionment.

[CRE Primer VI 50-53]

## ARINC Apportionment Technique.

- The failure rate for the system is first determined and then previous history or other estimation methods are used to provide a weighing ( $w_i$ ) for each subsystem to determine what the individual subsystem failure rate must be to achieve the system reliability requirement. The methods for calculating the weighting factors.

Step 1. Using available information or other estimation techniques, estimate the subsystem failure rate  $\lambda_i$  for each subsystem.

Step 2. Calculating the weighing ( $w_i$ ) for each subsystem as a proportion of the subsystem failure rate to the entire system failure rate.

$$\cdot w_i = \frac{\lambda_i}{\sum_{i=1}^n \lambda_i} \quad i = 1, 2, 3, \dots, n \quad \sum_{i=1}^n w_i = 1$$

Step 3. The subsystem failure rate requirement is then calculated.

$$\cdot \lambda_i = w_i \lambda_{system}$$