Paper Number 04M-105

Stochastic Approach for Vehicle Crash Models

David S. Riha
Southwest Research Institute

Joseph Hassan, Marlon Forrest and Ke Ding
Chrysler Group, DaimlerChrysler Corporation

ABSTRACT

This paper describes the development of a mathematical model capable of providing realistic simulations of vehicle crashes by accounting for uncertainty in the model input parameters. Advanced and efficient probabilistic and reliability analysis methods are coupled with well-established, high fidelity finite element and occupant modeling software to predict the reliability of vehicle impact scenarios.

The NESSUS probabilistic analysis software was used as the framework for a stochastic crashworthiness FE model. The LS-DYNA finite element model of vehicle frontal offset impact and the MADYMO model of a 50th percentile male Hybrid III dummy were integrated with NESSUS to comprise the crashworthiness characteristics. Response quantities from the models were used to define four occupant injury acceptance criteria and six compartment intrusion criteria. These ten acceptance criteria were used as events in a probabilistic fault tree to compute the overall system reliability of the impact scenario. A response surface model was developed for each acceptance criteria to facilitate the probabilistic analysis and vehicle design tradeoff studies.

NESSUS was used to compute the reliability of each acceptance criteria and the system reliability by combining all acceptance criteria events into a probabilistic fault tree. A redesign analysis was performed using the computed probabilistic sensitivity factors to direct design changes. These sensitivities were used to identify the most effective changes in model parameters to improve the reliability. A redesign using 11 design modifications was performed that increased the original reliability from 23% to 86%. Several of the design changes include increasing the rail material yield strength and reducing its variation, reducing the variation of the bumper and rail installation tolerances, and increasing the rail weld stiffness and reducing its variation. The NCAP star rating was also computed for the original and final designs as another measure of vehicle performance.

Finally, the response surface models were compared to the actual numerical models to verify their accuracy. Accuracy, benefits and limitations of the response surface approach for crashworthiness models is also discussed. The results show that major reliability improvements for occupant injury and compartment intrusion can be realized by certain specific modifications to the model input parameters. A traditional (deterministic) method of analysis would not have suggested several of these modifications.

INTRODUCTION

Numerical methods are widely used for the analysis of the impact behavior. The most commonly used computational methods for vehicle impact analysis is the rigid body formulation implemented in codes such as MADYMO and PAM-SAFE and the finite element method implemented in codes such as LS-DYNA, PAM-CRASH and RADIOSS. Using these codes, designs of almost any arbitrary complexity and physical behavior can be investigated for their impact responses. This way, a large number of variations and design modifications can be compared effectively. From the results of computational simulations, conclusions can be drawn to develop understanding of the impact response. The determination of such changes is usually driven by the experience of the engineer.

A typical CAE simulation for crash and safety analysis requires the use of a wide range of input parameter classes such as dimensions, material properties, geometrical constraints, boundary approximations and numerical assumptions. Geometry is obtained from a "typical" test setup, which is considered to be an accurate representation of the test being simulated. However, a single impact test is insufficient to capture the relevant physics of the characteristic response of a vehicle and occupants subjected to impact conditions.

Metallic material properties used in crash analysis suffer from uncertainty in characteristics such as yield, strength, strain hardening and others. Typical scatter values are between 5 to 10%. Uncertainty is even higher for properties such as strain rate, foam material definitions and weld characteristics.

With regard to reliability, the design procedure usually entails comparing analysis results to those of a design guide, or for the case of safety and crash, some NHTSA requirement. The problem with this approach is that it does not take into account (in a quantifiable manner) the fact that there is inevitably some element of uncertainty in the basic design parameters, such as material properties, tolerances, and loadings. In this work, the influence of parameters such as uncertainty in weld quality (stiffness, failure strength), uncertainty in various material properties (yield, ultimate strength, strain hardening), uncertainty in local thickness of stamped parts and finally imperfections due to actual assembly processes, were used as input variables.

STOCHASTIC ANALYSIS OF VEHICLE IMPACT SIMULATION

Uncertainties and non-deterministic behavior exist in all physical processes that are known as the core applications of occupant response simulation using numerical formulations such as the case in LS-DYNA [1] and MADYMO [2]. Car crash and occupant safety are applications of impact-type problems that are highly non-deterministic, as such repeated collision tests of the same vehicle type will always lead to different results.

The idea behind probabilistic structural analysis is to use the information about the probability of random variables, along with the structural behavior, in order to quantify the scatter in the structural response. The result from the analyses is a distribution in the form of a Probability Distribution Functions (PDF) and Cumulative Distribution Function (CDF), of the structural response. Thus, the analysis gives a more complete picture of the actual simulation. With this method, the probability of achieving a certain level of structural response can be computed. While the conventional Probabilistic Structural analysis (PSA) considers the uncertain quantities as *random variables*, in Stochastic Structural analysis (SSA) they are modeled as *random fields,* as such, they are considered as sequences of random variables spread over the volume of the structure object with a defined correlation structure. Therefore, the consideration of randomness in this regard is more elaborate since it is assumed that the parameters of the model have a random spatial variation [3].

The concept of a stochastic crash simulation can be explained due to the fact that the problem comprises a set of stochastic structural parameters, stochastic external forces and boundary conditions and, finally, the stochastic output variables such as displacements, accelerations, and internal energies. The stochastic crash problem may be stated, in general terms as follows: *Given the Probability Density Functions (PDFs) of the stochastic structural parameters, external forces, boundary and initial conditions, determine the corresponding PDFs of the output variables.*

In generic mathematical terms, crash is a dynamic phenomenon that may be formally described by a set of nonlinear first order vector differential equations in the form;

$$x = f\,(x,F,p)$$

$$y = g\,(x,p)$$

where $x \in R^N$ is the state vector of displacements and velocities, $F \in R^p$ represents the stochastic external forcing terms, $p \in R^n$ is a vector of stochastic structural parameters and $y \in R^q$ the measurement vector (e.g. accelerations, strains, etc.). A classical problem in stochastic mechanics is the computation of the Probability Distribution Functions (PDFs) of the output variables given the PDFs of the input variables. This can be accomplished by knowing the joint density function of all the random variables implied in a specific problem. The most widely used technique to estimate the required joint density function is the Nataf model, which uses a multidimensional Gaussian distribution with correlation coefficients modified according to the non linear transformation linking the given marginal and the Gaussian densities. The corresponding samples of the correlated variables can then be generated by means of this approximate distribution.

The objective of this work was to develop a mathematical model capable of providing realistic simulations of vehicle crashes by accounting for uncertainty in the model input parameters. The stochastic crashworthiness model consists of ten acceptance criteria. The performance models for each acceptance criteria are defined using stochastic based software called NESSUS [4,5]. NESSUS is an advanced probabilistic analysis code developed by Southwest Research Institute. The LS-DYNA explicit dynamic finite element software is used to compute the time-dependent structural response of the vehicle due to frontal impact loads. Response quantities such as acceleration and displacement are used for the compartment intrusion performance measures and as input to the MADYMO occupant response program to compute occupant injury measures. The LS-DYNA finite element model and the MADYMO model of a 50th percentile male Hybrid III dummy are integrated with NESSUS to comprise the crashworthiness model. This integrated model provides an automatic and flexible uncertainty analysis procedure to allow the modeler the ability to simulate uncertainties in any LS-DYNA and MADYMO input variables. The modeler can easily select any relevant model responses to define a wide range of acceptance criteria measures. The problem setup, analysis and interpretation of results are all handled through the NESSUS graphical user interface.

STOCHASTIC MODEL

For the analysis reported herein, ten acceptance criteria are used to evaluate the crashworthiness of the vehicle-to-vehicle frontal offset impact model. These criteria include four occupant injury and six compartment intrusion measures.

Uncertainty inputs to the model consist of 20 random variables. These random variables include parameters that define key energy absorbing components of the vehicles such as material properties for bumpers and rails, test environment uncertainties such as impact velocity and angle, and manufacturing variations in the form of rail and bumper installation parameters. Each of these random variables is characterized by a statistical distribution defined from manufacturing data, literature and/or expert opinion.

A probabilistic system reliability analysis is required to correctly evaluate the vehicle performance, especially for computing the probabilistic sensitivity factors at the system level for redesign analysis. For example, certain parameters such as stiffness/strength parameters can improve the reliability for compartment intrusion performance measures, but they may be detrimental to the crash pulse attenuation seen by the vehicle's occupant. The probabilistic system model correctly accounts for events with common variables (correlated events) and thus correctly identifies the important variables on the system level.

The probabilistic fault tree capability was used to develop the system reliability model. System reliability for this analysis is the probability of meeting the combination of all ten acceptance criterion. The model consists of the ten acceptance criteria defined by the RSM performance functions. The probabilistic fault tree analysis was also used to compute probabilistic sensitivity factors. These sensitivities are the derivatives of the probability of failure with respect to the random variable parameters (mean and standard deviation). The magnitude and direction of the sensitivities indicate the variables that contribute most to the reliability. For the original design it was determined that the yield stress of the small vehicle rail material contributes most to the reliability of the design.

To facilitate the probabilistic analysis and a reliability improvement study, a response surface model (RSM) is developed for each acceptance criteria performance model. The RSM approach aids in reducing the number of required LS-DYNA and MADYMO analyses and provides a fast running function that can be used for design trade off studies.

COMPONENT PROBABILISTIC AND RELIABILITY ANALYSIS

One of the key outputs of this approach is the cumulative distribution function (CDF) of system performance (e.g. stress or strain). The CDF is the probability that the performance value is less than or equal to a specified value. Traditional reliability analysis involves computing the probability that the stress (S) will exceed the strength (R), or P[g<0] where

$$g = R - S$$

R and S may be complex models involving other random variables such as $R(X_i)$ and $S(Y_i)$. The g-function is formulated such that $g=g(X_i,Y_i)$ and thus correctly accounts for possible correlation between the stress and strength parts of the performance measure (i.e., common random variables in R and S). This approach provides a general formulation of the g-function that allows different analysis codes and analytical functions to be linked together in a hierarchical fashion. For example, a stress or strain from a finite element analysis can be used with a fatigue life equation or S-N curve to define the performance of a structure.

SYSTEM RELIABILITY ANALYSIS

Most engineering structures have multiple conditions that are evaluated for acceptable performance. These events are defined by different acceptable conditions of a component or multiple component/conditions of a system. The nonperformance of one or a combination of events can lead to nonperformance of the system. System reliability considers acceptable performance of multiple components of a system and/or acceptable conditions of a component. Many options are available for predicting system reliability including bounding methods, boolean combinations of the event probabilities (with and without assuming the events are independent) and brute force Monte Carlo simulation [6,7]. System reliability in NESSUS is currently addressed using a probabilistic fault tree analysis (PFTA) method [8,9].

System reliability is defined through a fault tree by defining the bottom events and their combination with "AND" and "OR" gates. Each bottom event considers a single performance event (component reliability) and can be defined by a finite element model and performance function or as an analytical equation. An example of a fault tree for a three-event system is shown in Figure 1.

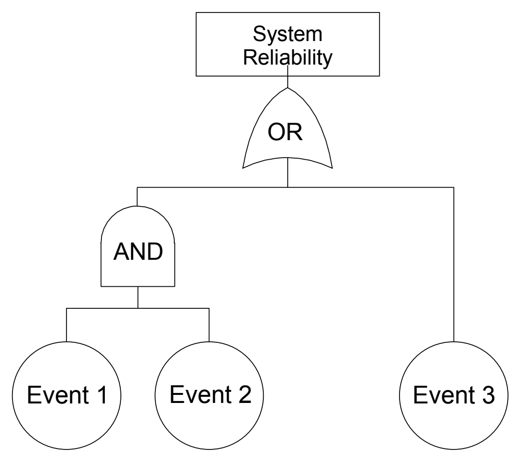

Figure 1. Example of a Fault Tree for System Reliability Analysis in NESSUS

RESPONSE SURFACE METHODS

The response surface approach for probabilistic analysis can be used to replace a computationally intensive function evaluation with a fast-running polynomial expression. Several components are required for performing probabilistic analysis using a response surface as shown in Figure 2.

- Experimental design
- Model fitting/ANOVA
- Probabilistic Analysis (usually simulation)

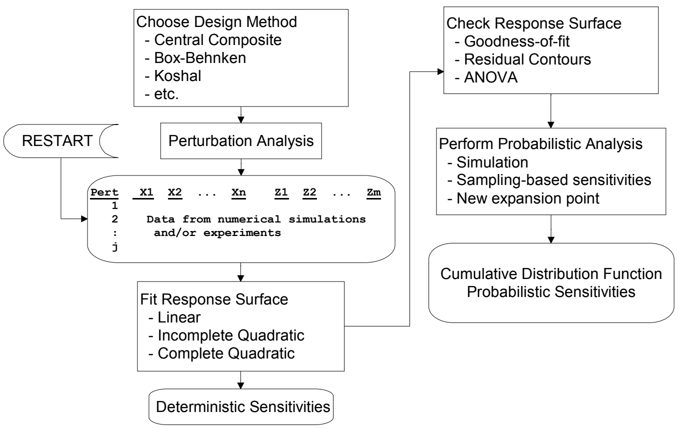

Figure 2. Probabilistic Analysis using a Response Surface Model

Response surface models (RSM) have some unique advantages for performing probabilistic analysis:

- Easy transfer of information between modelers
- Best suited for estimating mean and standard deviation
- Can incorporate test and numerical data
- Quickly generate accurate g-functions for system analysis
- Reduced number of function evaluations in some cases
- Fast running function that can be used for design tradeoff studies

While the RSM is a powerful tool for the probabilistic analyst, there are some disadvantages to its use:

- May not be accurate for small probabilities
- Difficult to assess error without rerunning the original function in the design region
- Typically not as efficient as AMV+

PROBABILISTIC SENSITIVITY FACTORS

As part of the probabilistic analysis, the probabilistic sensitivity measures are computed in the form of the derivatives of the probability of failure with respect to the random variable parameters:

$$\frac{\partial p}{\partial \mu_i} \text{ and } \frac{\partial p}{\partial \sigma_i}$$

Where i is the particular random variable, p is the probability, and μ and σ are the mean and standard deviation of the i^{th} random variable respectively.

MONTE CARLO SIMULATION

Simulation methods repeatedly evaluate the performance function to generate a sample of the response. The response samples are used to estimate the mean, standard deviation or other statistics. For simulation methods, there is no requirement that the response function be well behaved or that there is any practical restriction on the number of random variables as there are with most probable point based methods. The Monte Carlo simulation method uses random samples from the probability distributions of each random variable and computes the response. The reliability is computed by counting the number of acceptable evaluations of the performance function. Other information can be obtained such as the empirical cumulative distribution function (CDF), probability distribution of the response, and moments of the response. The method is exact as the number of simulations approaches infinity. The major disadvantage to Monte Carlo simulation is that the analysis can be time consuming for complex deterministic models. Even for fast running models the analysis can require a large number of samples for large or small reliabilities as indicated using a sampling error equation

$$\%error = 100 \cdot \Phi^{-1}[1-\alpha/2]\sqrt{\frac{1-p}{k \cdot p}}$$

In this equation, (1-α) is the confidence level, p is the probability, and k is the required sample size. The equation states that there is a (1-α) chance that the %error in the estimated probability will be less than that given in the equation. This equation is valid for probabilities less than 0.5. If the probability is greater than 0.5 then 1- p is used to evaluate the error.

SYSTEM RELIABILITY ANALYSIS FOR INDEPENDENT EVENTS

Defining the event as a desirable outcome, then the intersection of all events produces the system reliability as:

$$R = P(E_1 \cap E_2 \cap E_3 ... \cap E_n)$$

where E_i is the occurance of the i^{th} event. In many system reliability analyses, the correlation between events is ignored and the events are assumed to be independent. If the events are assumed independent, then the system reliability is the product of the independent event reliabilities defined as:

$$P(E_1 \cap E_2 \cap E_3 ... \cap E_n) = \prod_{i=1}^{n} P(E_i)$$

DETERMINISTIC MODEL

The crashworthiness model used in this analysis simulates vehicle-to-vehicle frontal offset impact. The vehicles include a small car and larger SUV style vehicle as shown in Figure 3. The LS-DYNA finite element model used in this analysis was built by the National Crash Analysis Center (NCAC). It consists of over 250,000 nodes and 240,000 elements and was analyzed using LS-DYNA version 960 on SGI and HP parallel platforms. The analysis time was approximately 30 CPU hours using 8 processors.

The LS-DYNA model is used to compute the structure responses of the vehicle such as accelerations and displacements. These response quantities are used for the compartment intrusion performance measures and as input to MADYMO to compute occupant injury measures. The LS-DYNA finite element model and the MADYMO model with a 50th percentile male Hybrid III dummy are integrated with NESSUS to comprise the crashworthiness model. This integration framework is shown in Figure 4.

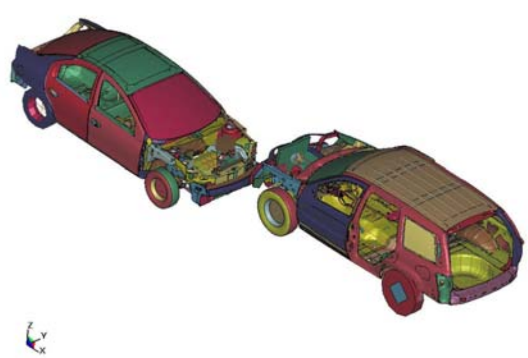

Figure 3. Vehicle-to-Vehicle Frontal Offset Crash Simulation Model.

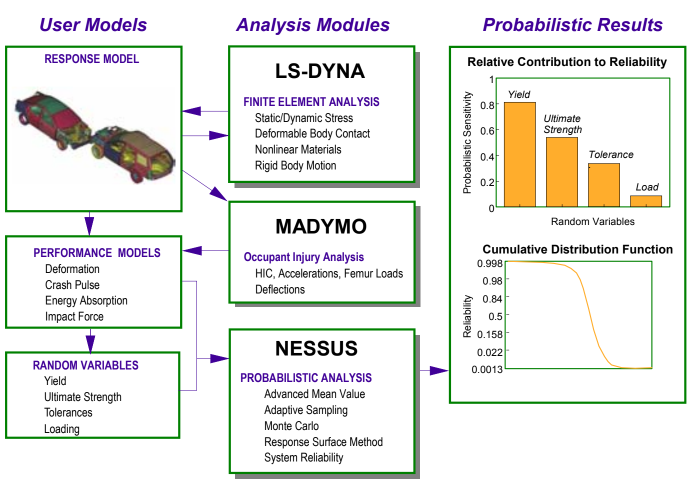

Figure 4. Stochastic Crash Simulation Framework

RANDOM VARIABLES

Uncertainties of several parameters in the FE model are defined for controlled test conditions. The large vehicle is at rest at a specified location. The small car is attached to a guide and is brought to a certain impact velocity. The velocity direction along the guide is fairly certain but the attachment of the vehicle to the rail and location of the large vehicle introduces uncertainty. The test setup is shown in Figure 5. Other random variables include uncertainties that arise in the manufacturing process and inherent uncertainty of material characteristics.

The distributions for parameters that affect the geometry are based on design/manufacturing tolerances. These tolerances are defined as ±3 standard deviations from the mean value for a process that is in statistical control. Table 1 lists the random variable descriptions followed by the probability distribution definitions in Table 2 used in the model.

SMALL VEHICLE RANDOM VARIABLES

RAIL CHARACTERISTICS

Vehicle rails are critical components to the energy absorbing characteristics of the vehicle. Uncertainties in the rail material properties such as yield strength, elastic modulus, metal thickness, and weld connections are considered to be the main parameters driving the energy absorption. The yield strength of high strength steel for rail material (YIELDCAR) is modeled as a random variable as well as the elastic modulus (EMODCAR). Rail thickness (RAILTCAR) is also modeled as a random variable. Variations in sheet metal parts occur from the manufacturing and forming processes. The left and right rails are generally manufactured using the same lot of material and at the same time. Therefore the left and right rail thickness variables are modeled as a single random variable (fully correlated). Uncertainty also exists in the attachment of the rails during manufacturing. The right (RTOLCARR) and left rail assembly tolerances (RTOLCARL) are modeled as random variables. The uncertainty is modeled as shown in Figure 6 where each rail may vary in the Z-direction of the model. Left and right small car rails with assembly uncertainty are modeled as two random variables. Variations in the rail assembly tolerance are modeled by shifting the rail in the Z-direction. The uncertainty of each rail placement is independent and based on design tolerances.

The rails are attached to the vehicle using spot welds. The number, size and quality of the welds affect the vehicles energy absorbing characteristics. The weld stiffness for the front rail (FRWSTIFF), front left rail (LRWSTIFF), and front right rail (RRWSTIFF) were considered as independent random variables. The same distribution (lognormal) and coefficient of variation as the weld diameters were used for the weld stiffness.

Table 1. Random Variable Description

	Description	Name
Small Vehicle		
1	HSS yield strength	YIELDCAR
2	Elastic modulus	EMODCAR
3	Foam density	FDENSCAR
4	Foam ultimate strength	FOAMUCAR
5	Right rail assembly tolerance	RTOLCARR
6	Left rail assembly tolerance	RTOLCARL
7	Impact velocity	IMPVEL
8	Impact angle	IMPANGL
9	Bumper height	BUMPER
10	Tire height	TIRE
11	Front rail weld (effective plastic strain)	FRWSTIFF
12	Front left rail weld (effective plastic strain)	LRWSTIFF
13	Front right rail weld (effective plastic strain)	RRWSTIFF
14	Rail thickness	RAILTCAR
Large Vehicle		
15	Foam density	FDENSTRK
16	Foam ultimate strength	FOAMUTRK
17	Right rail assembly tolerance	RTOLTRKR
18	Left rail assembly tolerance	RTOLTRKL
19	Rail thickness	RAILTTRK
20	Overlap	OVERLAP

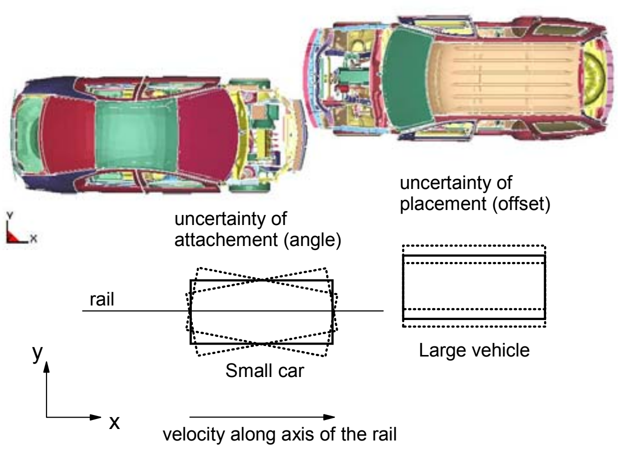

Figure 5. Model Setup

Figure 6. Left and right small car rails with assembly uncertainty are modeled as two random variables.

Table 2. Random Variables Definitions

	Name	Mean	Standard Deviation	Units	Dist.
Small Vehicle					
1	YIELDCAR	275	15.125	N/mm^2	Log.
2	EMODCAR	210000	10500	N/mm^2	Normal
3	FDENSCAR	8.00E-10	1.2E-10	Tonne/mm^3	Normal
4	FOAMUCAR	25	3.75	N/mm^2	Normal
5	RTOLCARR	0[1]	4.23333	mm	Normal
6	RTOLCARL	0[1]	4.23333	mm	Normal
7	IMPVEL	15643	156.43	mm/sec	Normal
8	IMPANGL	0	3.3333	degrees	Normal
9	BUMPER	0[1]	4.23333	mm	Normal
10	TIRE	0[1]	4.23333	mm	Normal
11	FRWSTIFF	1.33E-03	1.33E-04	-	Log.
12	LRWSTIFF	1.33E-03	1.33E-04	-	Log.
13	RRWSTIFF	1.33E-03	1.33E-04	-	Log.
14	RAILTCAR	1.9	0.0342	mm	Normal
Large Vehicle					
15	FDENSTRK	4.95E-11	7.4184E-12	Tonne/mm^3	Normal
16	FOAMUTRK	210	31.5000	N/mm^2	Normal
17	RTOLTRKR	0[1]	4.23333	mm	Normal
18	RTOLTRKL	0[1]	4.23333	mm	Normal
19	RAILTTRK	1.73	0.03114	mm	Normal
20	OVERLAP	0[1]	0	mm	Normal

[1] mean value is relative to the nominal installation position

BUMPER CHARACTERISTICS

The bumper foam density (FDENSCAR) and ultimate strength (FOAMUCAR) are modeled as random variables. The uncertainty of bumper height (BUMBER) is also included in the model. Uncertainty in bumper height arises during the attachment process in manufacturing and is modeled by moving the bumper nodal coordinates in the Z direction. The bumper is depicted in Figure 7.

VEHICLE HEIGHT

Another uncertainty is the vertical height of the vehicle that arises from tire inflation (TIRE). This uncertainty is modeled by changing the Z coordinate of the small vehicle with respect to the larger vehicle as shown in Figure 8. This uncertainty is based on design tolerances and test procedure requirements (±0.5 inches).

TEST PARAMETER UNCERTAINTIES

Several parameters cannot be controlled completely in a crash event and can be expected to lead to uncertainty in the crashworthiness performance. The impact velocity (IMPVEL) is modeled as random variable and the uncertainty is defined by the test requirements (±3%). The impact angle (IMPANGLE) is another uncertain parameter in the test and also defined by the test requirements (±10º). The impact angle variation is simulated about the small car center of gravity as shown in Figure 9.

LARGE VEHICLE RANDOM VARIABLES

All acceptance criteria for this model are for the small vehicle. However, uncertainties of several critical energy absorbing parameters in the large vehicle were also considered to determine their contribution to the event. These variables include the bumper foam density (FDENSTRK) and ultimate strength (FOAMUTRK), the left and right rail assembly tolerances (RTOLTRKL, RTOLTRKR), and the rail thickness. The distribution definitions for these parameters were obtained from supplier's data.

ACCEPTANCE CRITERIA

Ten acceptance criteria and the NCAP star rating criteria were used to evaluate the crashworthiness of the vehicle-to-vehicle frontal offset impact test. All acceptance criteria are evaluated for the small vehicle.

<u>Occupant and Compartment Intrusion Acceptance Criteria</u>

The criteria include four occupant injury and six compartment intrusion measures as listed in Table 3. Injury measures are based on a 50th percentile male Hybrid III dummy in the driver seat modeled with MADYMO.

The performance of each acceptance criteria is defined by LS-DYNA or MADYMO computed quantities. Information describing each of these performance measures is listed in Table 4.

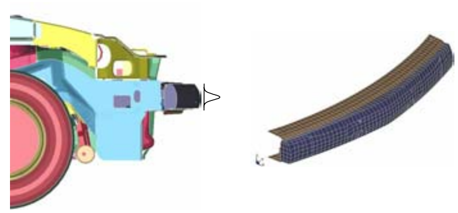

Figure 7. Bumper height assembly tolerance

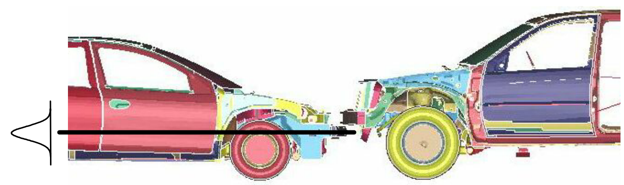

Figure 8. Tire height uncertainty is modeled by moving the small car nodal coordinates in the Z direction (distribution is not to scale)

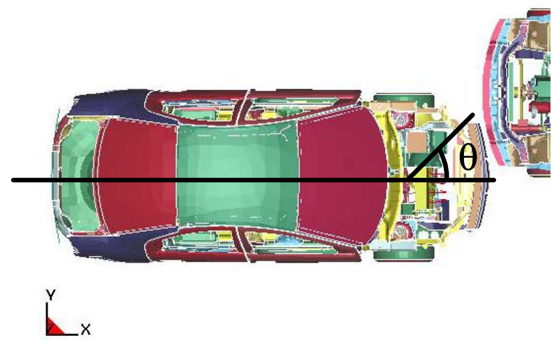

Figure 9. Impact angle uncertainty is modeled by rotating the small car nodal coordinates around the CG.

Table 3. Acceptance Criteria for the Crashworthiness Model

	Acceptance Criteria	Assessment Values
Occupant Injury		
1	Head Injury Criteria (HIC36)	1000 (36 msec clip)[1]
2	Chest acceleration (g's)	60g[1,2]
3	Chest deflection	64 (mm)[2]
4	Femur axial load	10000 (N)[1]
Passenger Compartment Intrusion		
5	Footrest intrusion	150 mm
6	Average of left/center/right toe pan deflection	150 mm
7	Brake pedal location	150 mm
8	Average of left/right instrument panel (IP) deflection	100 mm
9	Door aperture closure (measured at beltline on struck side)	100 mm
10	Engine displacement relative to left B-pillar	200 mm

[1]FMVSS 208

[2]"Frontal Offset Crashworthiness Evaluation: Guidelines for Rating Structural Performance," Insurance Institute for Highway Safety, April 2002.

Table 4 LS-DYNA and MADYMO Model Entities that Define the Acceptance Criteria

Acceptance Criteria	MADYMO Response	LS-DYNA Response
HIC	HIC	Negative of X acceleration[1]
Chest Acceleration	Chest g's	Negative of X acceleration[1]
Chest Deflection	Chest Deflection	Negative of X acceleration[1]
Femur Axial Load	Right Femur Compression	Negative of X acceleration[1]
Footrest Intrusion	N/A	X-disp. B-pillar X-disp. footrest
Toe Pan Deflection	N/A	X-disp. B-pillar X-disp. right toe pan X-disp. center toe pan X-disp. left toe pan
Brake Location	N/A	X-disp. B-pillar X-disp. Brake location
Instrument Panel Deflection	N/A	X-disp. B-pillar X-disp. left instr. panel X-disp. right instr. panel
Door Aperture Closure	N/A	X-disp. B-pillar X-disp. forward door frame X-disp. rear door frame
Engine Displacement	N/A	X-disp. B-pillar X-disp. Engine

[1] crash pulse

NCAP VEHICLE STAR RATING CRITERION

The New Car Assessment Program (NCAP) vehicle star rating value is also evaluated to determine the star rating for the design. The star rating is used to express the chance of incurring a serious injury in the event of a crash. In the explanation of ratings listed in Table 5, a serious injury is one requiring immediate hospitalization and may be life threatening.

Table 5. NCAP Vehicle Star Rating System

Rating	Probability of Serious Injury
★★★★★	0.0 < P ≤ 0.1
★★★★	0.1 < P ≤ 0.2
★★★	0 .2< P ≤ 0.35
★★	0.35 < P ≤ 0.45
★	0.45 < P

The NHTSA (Docket No. 97-29) computations of the probability for the head injury criteria (HIC) and the chest acceleration (CHEST_G) are defined by:

$$P_{head} = \frac{1}{1+e^{5.02-0.00351*HIC}}$$

$$P_{chest} = \frac{1}{1+e^{5.55-0.0693*CHEST\_G}}$$

The nominal values for HIC and CHEST_G are used for these probability calculations. The combined probability of serious injury for both the head and chest criteria is given by the system probability using the independent event assumption:

$$P_{combined} = P_{head} + P_{chest} - P_{head} \cdot P_{chest}$$

PROBABILISTIC CRASHWORTHINESS MODEL

Probabilistic analysis algorithms require response solutions for specific combinations of the random variable values. These perturbed responses created by changing the variables in the response models are used for random sampling or may be used for finite difference approximations of gradients used in fast probability integration methods.

The overall probabilistic crashworthiness model is defined by NESSUS, LS-DYNA and MADYMO as in Figure 4. The combination of these models provides an automatic procedure to relate changes in the random variables to the LS-DYNA and MADYMO input and to easily select model responses such as displacements used to define the acceptance criteria measures. The model allows the variation of any of the defined random variables.

A large portion of the development of this model involved defining how a change of each random variable is mapped to the LS-DYNA input. The mapping was defined and verified for each of the twenty variables. As an example, when the rail tolerance is perturbed, the resulting change in the LS-DYNA finite element model is automatically made by NESSUS based on the mapping definition (modifying the Z-coordinates of the nodes for the rail), the analysis rerun, and finally the response of interest extracted from the LS-DYNA results file. This procedure is fully automated in the developed model for all defined random variables.

ANALYSIS PROCEDURE

Solutions to nonlinear dynamic analyses are sensitive to changes in the geometry and material property parameters. In some cases these changes can be severe enough to cause numerical problems that terminate the analysis and, in others, the solution time step can become so small that the analysis time becomes prohibitive. The first attempt of computing the reliability for the footrest intrusion acceptance criteria using fast probability integration methods resulted in incomplete LS-DYNA solutions due to model sensitivity to these combined random variables. The cause of error of these cases was not investigated in detail. One suspected cause is the alignment of contact surfaces when geometry variables are changed. Remedying these cases would require portions of the finite element model to be regenerated resulting in a slightly different model and substantial development time. It was deemed more expedient to use an approach that would lend itself to less rigid requirements on the variable perturbations for this analysis.

A response surface approach was used to create functions defining each acceptance criteria measure because of the difficulty in automatically computing perturbed solutions required for fast probability integration methods. The response surface approach was also used since it would reduce the number of required LS-DYNA analyses and facilitates the redesign analysis. The response surface approach will not be as accurate as fast probability integration methods if the derived function does not represent the response over the design region. For this demonstration, the response surface approach was used to represent the general behavior of the acceptance criteria measures and also allowed the procedure to be demonstrated within a reasonable amount of computational effort. The accuracy of the response surface models should be checked in the design region for confidence in the probability predictions.

RESPONSE SURFACE MODEL GENERATION

Response surface models (RSM) were created for each of the ten acceptance criteria measures using a linear and quadratic term for each random variable (mixed terms were not obtained due to the computational expenses required to compute them). The approach was to obtain LS-DYNA and MADYMO solutions for each random variable perturbed ±2 standard deviations from the mean value. For several random variables only one perturbed solution was obtained. For these cases, only the linear term is included in the regression model.

Large vehicle rail thickness (RAILTTRK) was perturbed ±11.111 standard deviations. This change resulted from an improved definition of the standard deviation for this variable after the LS-DYNA analyses were complete. The perturbed responses for all acceptance criteria measures indicated that the change in the response was small and fairly linear and thus sufficient for the response surface models. Table 6 lists the perturbations used to create the response surface models for each acceptance criteria.

Finally, it was noted that several runs had been completed where the rail weld stiffness was not included in the LS-DYNA model. It was found that including the weld stiffness did not affect the mean value solution of the acceptance measures by a significant amount. Therefore, rather than rerunning these cases at a considerable computational expense, the regression models were corrected using a first order approximation (a ratio of each acceptance measure computed with and without the weld stiffness). For probabilistic analysis, the important aspect of the response surface is accurate derivatives in the response surface function. This first order correction provides reasonable accuracy of the derivatives for the response surface models.

Table 6. Perturbations used to Create Response Surface Models.

Variable	Pert 1	Pert 2
RTOLCARR	2	2
IMPVEL	2	-2
BUMPER	-2	-2
TIRE	2	2
RAILTTRK	11.111	-11.111
IMPAN	2	-2
YIELDCAR	0.331	-2
EMODCAR	1.714	-2
FDENSCAR	-2	-2
FOAMUC	2	-2
FOAMUTRK	2	-2
RTOLTRKR	2	-2
RTOLTRKL	2	-2
FRWSTIFF	2	-2
RRWSTIFF	2	-2
RAILTCAR	2	-2

A stochastic model was defined that included each of the acceptance criteria responses. Each response surface model was generated by selecting the appropriate response computed with the LS-DYNA and MADYMO analysis programs.

SYSTEM MODELS

A system reliability analysis is critical to correct evaluation of the vehicle performance especially for identifying the probabilistic sensitivity factors at the system level for redesign analysis. It is expected that certain parameters such as stiffness/strength can improve reliability for compartment intrusion performance measures but may be detrimental to the crash pulse attenuation as measured at the vehicle occupant. The system model correctly accounts for events with common variables (correlated events) and thus correctly identifies the important variables on the system level. System reliability for the model was computed using a probabilistic fault tree analysis (PFTA) method using the fault tree shown in Figure 10. The performance of each event is modeled using a response surface model. Monte Carlo simulation is used to compute the system reliability since each performance function is defined by a fast running response surface model. The system reliability is also computed using the individual event reliabilities based on the independent event assumption described earlier.

BASELINE RESULTS:

The system reliability was computed using the Monte Carlo simulation method in NESSUS with 100,000 samples. The computed system reliability for the original design is 23% (1.1% error, 95% confidence). The system reliability assuming independent events is 18%, which is a conservative estimate in this case. These results are summarized in Table 7. Finally an NCAP star rating of 4 was computed using the nominal values of the HIC and chest acceleration. These values are listed in Table 8.

A Monte Carlo analysis was performed for each event and the results are shown in Table 9. The error in the probability for a 95% confidence interval is also included in the table. The femur axial load acceptance criteria event has the lowest reliability followed by the HIC event and the door aperture closure event. All other acceptance criteria have relatively high reliability.

The probabilistic sensitivity factors computed are shown in Figure 11. These sensitivity factors define the parameters that can be most effectively modified to improve the reliability. From the figure, the nominal value of the yield strength of the small vehicle rail material can be most influential in increasing the reliability. The positive sign indicates that the reliability will increase if the yield strength is increased. The sign convention is opposite when the reliability is greater than 50%. The next most influential parameters are the nominal value of the front weld stiffness of the small vehicle rails followed by standard deviation (or scatter) of the small car yield strength. Several variables show negligible importance including the tire height and the foam properties for both vehicles.

The nominal value of the right car rail installation tolerance also shows some importance. This parameter is a center installation point and would not be considered for a design change without changes to the vehicle structure.

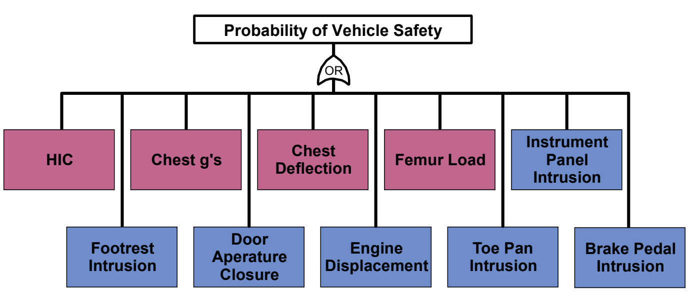

Figure 10. Probabilistic Fault Tree of the Crashworthiness Model

Table 7. System Reliability of the Original Design

System Reliability Approach	Reliability
NESSUS PFTA	23% (1.1% error)
Independent Events	18%

Table 8. NCAP Star Rating for Original Design

Design	HIC	Chest Acceleration	Combined Probability	Star Rating
Original	784	45 g's	17%	4

Table 9. Event Reliability for Original Design

Acceptance Criteria	Reliability	Samples (Error)
HIC	57.7910%	100,000 (0.7%)
Chest acceleration	92.2970%	100,000 (2.1%)
Chest deflection	99.9752%	2,000,000 (8.8%)
Femur axial load	46.4020%	100,000 (0.6%)
Footrest intrusion	99.9623%	2,000,000 (7.1%)
Toe pan deflection	100.0000%	2,000,000 (N/A)[1]
Brake pedal location	100.0000%	2,000,000 (N/A)[1]
Instrument panel def.	99.6870%	2,000,000 (2.5%)
Door aperture closure	72.6750%	200,000 (0.7%)
Engine location	99.6000%	2,000,000 (2.2%)

[1]NESSUS limit on samples and all samples resulted in a safe condition

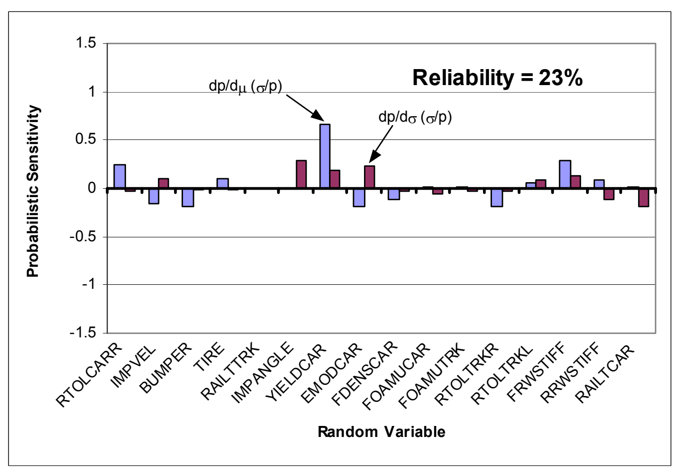

Figure 11. Probabilistic Sensitivity Factors for the Original Design.

ANALYSIS ITERATIONS:

The objective of the analysis is to provide a recommendation to improve the reliability of the small vehicle in a vehicle-to-vehicle frontal offset impact. The approach used is to rely on the probabilistic sensitivity factors to identify the dominant parameters (random variable mean and standard deviation) that will improve system reliability. Several restrictions were placed on the parameters that could be modified:

- Change only small car parameters.
- Parameters can only be changed by ±10% with exception of the weld stiffness. Changing the number of welds or weld diameter can increase/decrease the weld stiffness.
- Limit extrapolating the response surface models.

The following steps are performed for each parameter identified from the probabilistic sensitivity factors as candidates to improve the reliability:

- Modify the appropriated mean or standard deviation in the problem statement,
- Evaluate the reliability using Monte Carlo simulation,
- Evaluate the probabilistic sensitivity factors to find additional parameter candidates for reliability improvement if warranted, and
- Evaluate the acceptance criteria for event reliabilities once a model with acceptable system reliability is reached to identify the dominant events.

Table 10 lists the iteration history along with the justification for modifying each parameter. The table also lists the reference to the probabilistic sensitivity measures used to select the parameter to improve the reliability. The reliability and star rating improvements are shown in Figure 12. The system reliability for the final design is 86% (1.5% error) with a 5 star rating. The system reliability assuming independent events is 85%, which is a slightly conservative estimate for this case. These results are summarized in Table 11.

The parameters of the final iteration are listed Table 12. Finally, the event reliabilities were evaluated for the final configuration to identify the dominant acceptance criteria as listed in Table 13. The dominant event for the original design was the femur axial load acceptance criteria. The femur axial load also shows the lowest reliability for the final design but increased from a reliability of 46% to 93%.

Table 10. Redesign Iteration History Including Justification for Parameter Changes.

Iter.	Reference	Modification (Justification)	Reliability
0	Figure 11	Original design	23%
1	Figure 13	Increased small vehicle yield stress. (use a different material for the rails)	28%
2	Figure 14	Reduce COV of small vehicle yield stress to 4%. (improved quality control from supplier)	30%
3	Figure 15	Increase small vehicle weld stiffness. (larger weld diameter or increased number of welds)	40%
4	Figure 16	Increase front rail weld stiffness. (larger weld diameter or increased number of welds)	48%
5	Figure 17	Reduce COV of small vehicle yield stress to 2%. (improved quality control from supplier)	53%
6	Figure 18	Reduce front rail weld stiffness COV to 5%. (improved quality control of assembly line)	62%
7	Figure 19	Reduce rail thickness COV from 1.8% to 1% (improved quality control from supplier)	66%
8	Figure 20	Reduce front rail weld stiffness COV to 2%. (improved quality control of assembly line)	70%
9	Figure 21	Reduce COV of small vehicle yield stress to 1%. (improved quality control from supplier)	79%
10	Figure 22	Reduce COV of small vehicle foam material to 10%. (improved quality control from supplier)	83%
11	Figure 23	Tighten installation tolerances for bumper and rail. (improved quality control of assembly line)	86%

Table11. System Reliability of the Final Design

System Reliability Approach	Reliability
NESSUS PFTA	86% (1.5% error)
Independent Events	85%

Table 12. Random Variables (Final Design)

	Name	Mean	Standard Deviation	Units	Dist.
Small Vehicle					
1	YIELDCAR	280	2.8	N/mm^2	Log.
2	EMODCAR	210000	10500	N/mm^2	Normal
3	FDENSCAR	8.00E-10	8.00E-11	$Tonne/mm^3$	Normal
4	FOAMUCAR	25	2.5	N/mm^2	Normal
5	RTOLCARR	0	2.1167	mm	Normal
6	RTOLCARL	0	4.23333	mm	Normal
7	IMPVEL	15643	156.43	mm/sec	Normal
8	IMPANGL	0	3.3333	degrees	Normal
9	BUMPER	0	2.1167	mm	Normal
10	TIRE	0	4.23333	mm	Normal
11	FRWSTIFF	1.6E-3	3.2E-5	-	Log.
12	LRWSTIFF	1.33E-03	1.33E-04	-	Log.
13	RRWSTIFF	1.463E-03	7.315E-05	-	Log.
14	RAILTCAR	1.9	0.019	mm	Normal
Large Vehicle					
15	FDENSTRK	4.95E-11	7.4184E-12	$Tonne/mm^3$	Normal
16	FOAMUTRK	210	31.5000	N/mm^2	Normal
17	RTOLTRKR	0	4.23333	mm	Normal
18	RTOLTRKL	0	4.23333	mm	Normal
19	RAILTTRK	1.73	0.03114	mm	Normal
20	OVERLAP	0	0	mm	Normal

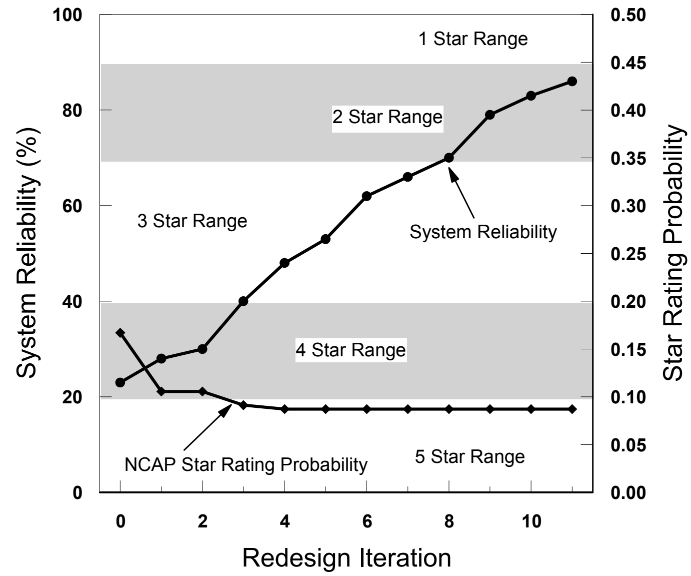

Figure 12. System Reliability Improvement

Table 13. Reliability for each Acceptance Criteria (Original and Final Designs).

Acceptance Criteria		Reliability		Samples (Error)
Description	**NESSUS Variable**	**Original Design**	**Final Design**	**Final Design**
HIC	g_hic	57.7910%	94.0120%	100,000 (2.4%)
Chest acceleration	g_cg	92.2970%	98.8240%	200,000 (4.0%)
Chest deflection	g_chestd	99.9752%	99.9999%	2,000,000 (196%)[1]
Femur axial load	g_femurl	46.4020%	92.9330%	100,000 (2.2%)
Footrest intrusion	g_fri	99.9623%	100.0000%	2,000,000 (N/A)[2]
Toepan deflection	g_tpd	100.0000%	100.0000%	2,000,000 (N/A)[2]
Brake pedal location	g_bpd	100.0000%	100.0000%	2,000,000 (N/A)[2]
Instrument panel def.	g_ipd	99.6870%	99.9719%	2,000,000 (8.3%)
Door aperture closure	g_dac	72.6750%	98.7460%	200,000 (3.9%)
Engine location	g_engd	99.6000%	99.9997%	2,000,000 (88%)

[1]NESSUS limit on samples. The reliability is ±error*(1-Reliability) resulting in the prediction well within an order of magnitude of (1-Reliability).
[2]NESSUS limit on samples and all samples resulted in a safe condition.

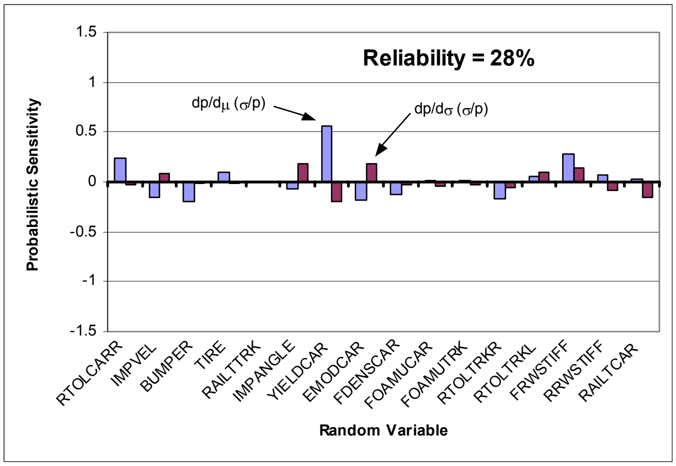

Figure 13. Probabilistic Sensitivity Factors for Iteration 1.

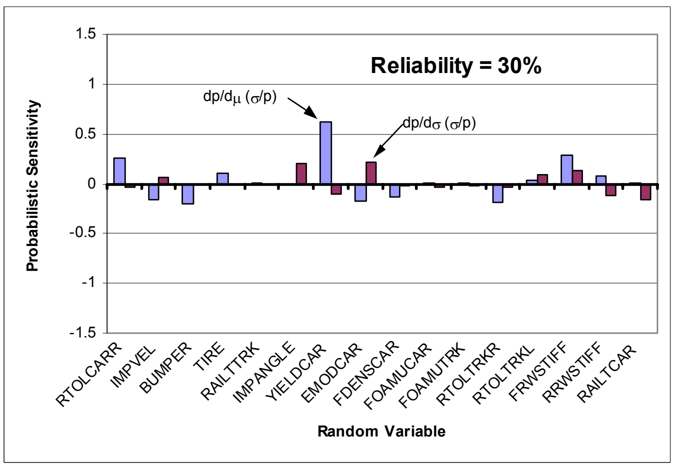

Figure 14. Probabilistic Sensitivity Factors for Iteration 2.

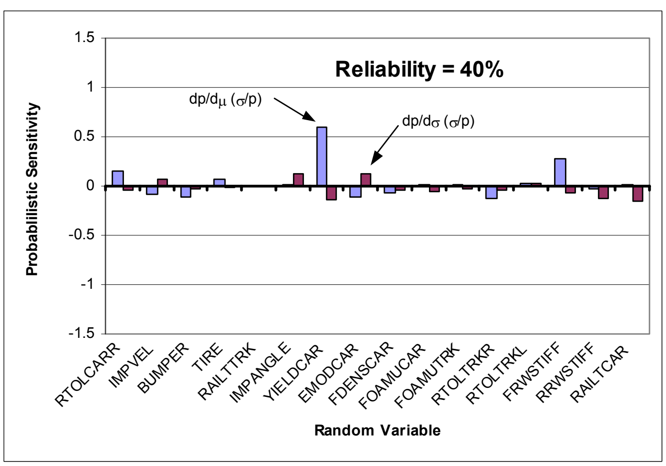

Figure 15. Probabilistic Sensitivity Factors for Iteration 3.

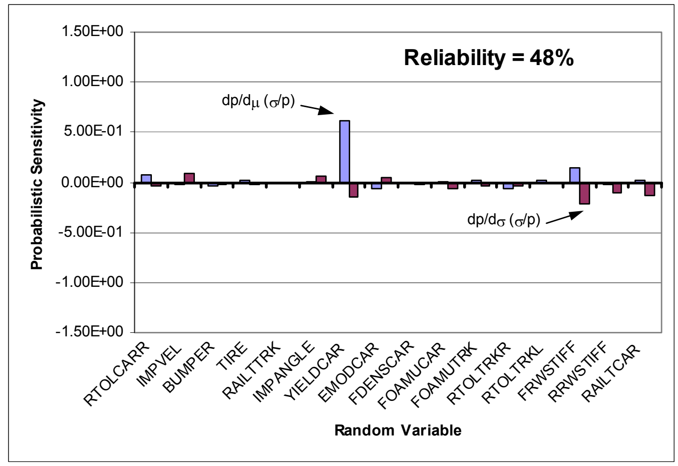

Figure 16. Probabilistic Sensitivity Factors for Iteration 4.

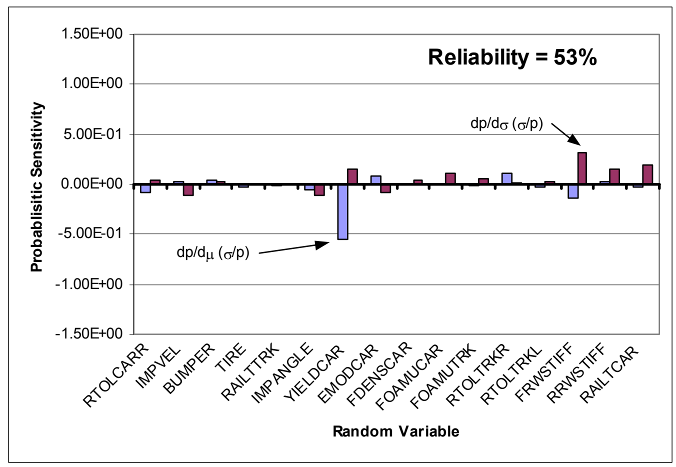

Figure 17. Probabilistic Sensitivity Factors for Iteration 5.

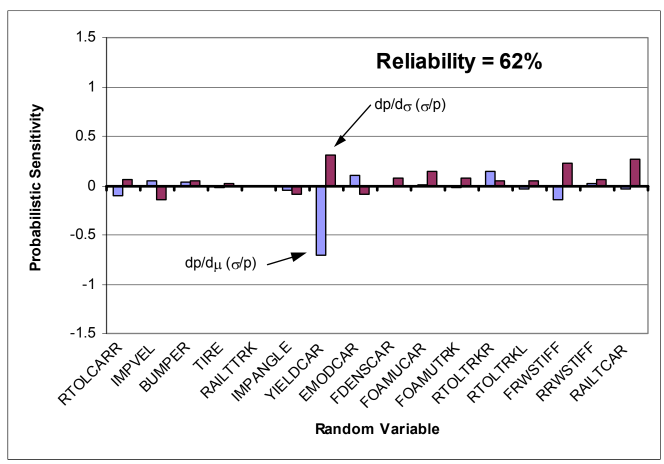

Figure 18. Probabilistic Sensitivity Factors for Iteration 6.

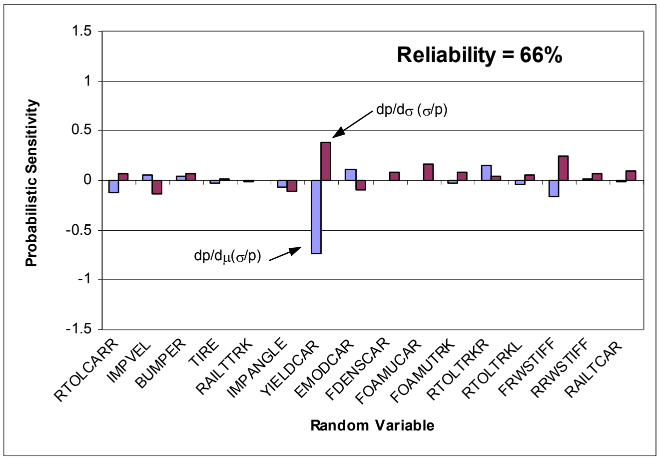

Figure 19. Probabilistic Sensitivity Factors for Iteration 7

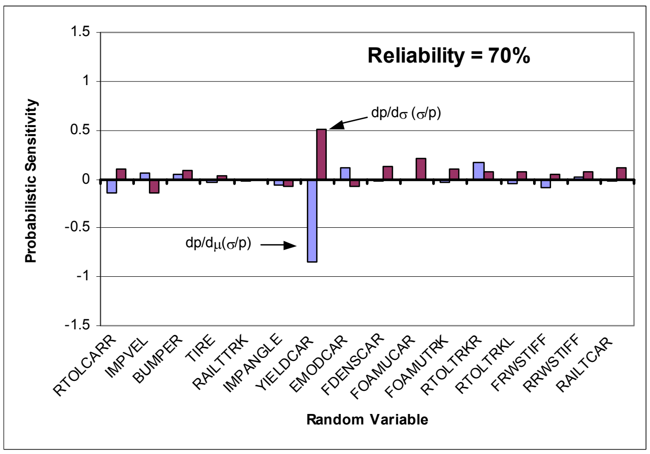

Figure 20. Probabilistic Sensitivity Factors for Iteration 8.

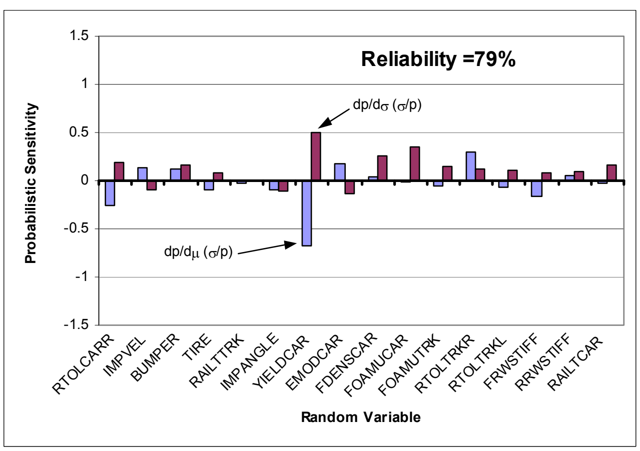

Figure 21. Probabilistic Sensitivity Factors for Iteration 9.

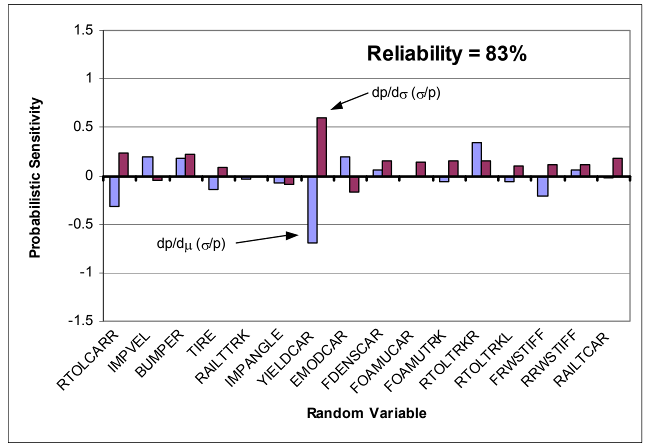

Figure 22. Probabilistic Sensitivity Factors for Iteration 10.

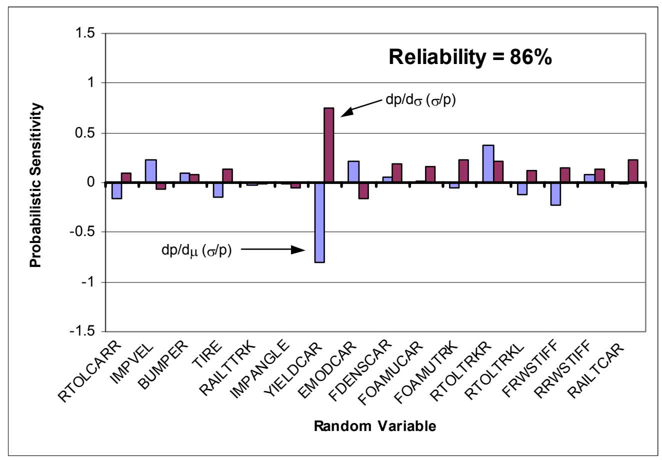

Figure 23. Probabilistic Sensitivity Factors for Iteration 11 (Final Design).

RESPONSE SURFACE VERIFICATION

Verification of the response surface in the area of interest is a critical step in the analysis. A simple verification can be achieved by rerunning the analysis codes using the design parameters obtained using the response surface method. Acceptance criteria computed using these re-runs are then compared by the values predicted by the RSM. While this approach does not guarantee the accuracy of the approach, it does provide design direction and establish the confidence in the proposed design changes.

The LS-DYNA and MADYMO models were analyzed for each acceptance criteria using the parameter values for the final design listed in Table 12. This step is used to determine the accuracy of the response surface models used for the reliability calculations. The results for each acceptance criteria are listed in Table 14. The table compares the improvement in the acceptance criteria based on both the actual model for both the original and final designs. The results from the actual model using the new design parameters are also compared to the RMS method and a percent error is listed. A negative error corresponds to a conservative prediction by the response surface model.

Table 14 indicates that in several cases the improvement as indicated by model re-runs are higher than the values predicted by RMS method. For example, HIC was found by the re-run model to be twice as improved as predicted by the RMS method. In some other cases the parameter predictions were found to be of very little difference. Out of the ten acceptance criteria, the approach resulted in an improvement of three acceptance criteria when compared to the original models. Because of the high reliability of the majority of the events, these slight increases of acceptance criteria are not expected to impact the system reliability. These are trade-offs for individual acceptance criteria reliability to improve the overall system reliability. However, the femur load and door aperture closure criteria need further investigation. These criteria exceed the deterministic threshold limits and were events that demonstrated lower reliability in the system analysis.

The NCAP star rating is listed in Table 15 to compare the response surface and actual models. The approximate response surface models provide a conservative estimate of the star rating.

The cause(s) of the discrepancies between the RMS and actual models have not been investigated as of the writing of this paper. However, some of the potential sources of error in the response surface models include:

1. Linear terms for several variables
2. No mixed terms
3. Second order function does not fit the true response
4. RSM is primarily a “local” or “near field” approach
5. Possible numerical errors in the models

Table 14. Comparison of the LS-DYNA and MADYMO Results (re-runs) with Response Surface Models.

Acceptance Criteria	Original Design Models	Final Design Models	RSM	Error
Footrest intrusion (mm)	7	9	15	-63%
Toe pan deflection (mm)	16	19	8	58%
Brake pedal location (mm)	12	14	7	47%
Instrument panel deflection (mm)	67	72	67	7%
Door aperture closure (mm)	100	106	76	29%
Engine location (mm)	156	161	141	12%
HIC	784	235	416	-77%
Chest acceleration (g’s)	45	41	41	2%
Chest deflection (mm)	61	60	50	15%
Femur load (N)	1052	10636	9858	7%

Table 15. NCAP Star Rating for Implicit Model

Design	HIC	Chest Acceleration	Combined Probability	Star Rating
Original	784	45 g’s	17%	4
Final (RSM)	416	41 g’s	8.7%	5
Actual Model	235	41 g’s	7.6%	5

CONCLUSION

A stochastic crashworthiness model was developed capable of providing simulations of vehicle crashes by accounting for the uncertainty of input parameters. The NESSUS probabilistic analysis software provided a framework in which the LS-DYNA model of vehicle frontal offset impact and the MADYMO model of a 50th percentile male Hybrid III dummy were combined to compute the crashworthiness characteristics. The model allows an automatic procedure to relate changes in the random variables to the LS-DYNA and MADYMO input and easily select model responses such as displacements used to define the acceptance criteria measures. The model allows the variation of any of the defined random variables and other model parameters can easily be included as random variables.

To facilitate the probabilistic analysis and the redesign analyses, a response surface model (RSM) was developed for each acceptance criteria performance model. The RSM approach aided in reducing the number of required LS-DYNA analyses and provided a fast running function that could be used for the design study tradeoff analysis. Based on the RSM models, the femur axial load acceptance criteria event was found to have the lowest reliability (46%) followed by the HIC event (58%) and the door aperture closure event (73%). A system reliability analysis was used to include the contribution of all acceptance criteria to correctly quantify the vehicle reliability and identify important parameters. The system reliability of the original design was computed as 23%. The NCAP star rating was also computed as another measure of vehicle performance. The original design predicted a 4 star rating.

A redesign approach was developed and performed using the probabilistic sensitivity factors. Eleven iterations were performed resulting in a system reliability of 86% (original design reliability was 23%). The design changes include increasing the rail material yield strength and reducing the variation, reducing the variation of the bumper and rail installation tolerances, and increasing the rail weld stiffness and reducing its variation, and reducing the variation of the foam properties of the small vehicle. It should be noted that evaluating a new design requires very little computational expense since the performance functions are defined by response surface models. A final evaluation of the reliability for each acceptance criteria in the new design was performed to identify the dominant criteria. The reliability of the original design was dominated by the femur load acceptance criteria (46%). The femur axial load also shows the lowest reliability for the final design but increased from a reliability of 46% to 93%.

A system reliability analysis is critical to correct evaluation of the vehicle performance especially for evaluating the probabilistic sensitivity factors at the system level for redesign analysis. Certain parameters such as stiffness/strength parameters can improve reliability for compartment intrusion performance measures but may be detrimental to the crash pulse attenuated to the vehicle occupant. The system model correctly accounts for events with common variables (correlated events) and thus correctly identifies the important variables on the system level.

The accuracy of the response surface models was evaluated by rerunning the LS-DYNA and MADYMO models using parameters for the new design. Out of the ten acceptance criteria, the approach resulted in an improvement of three acceptance criteria when compared to the original models. Because of the high reliability of the majority of the events, these slight increases of acceptance criteria are not expected to impact the system reliability. These are trade-offs for individual acceptance criteria reliability to improve the overall system reliability. However, the femur load and door aperture closure criteria need further investigation. These criteria exceed the deterministic threshold limits and were events that demonstrated lower reliability in the system analysis.

The developed models and redesign approached yielded an improved vehicle design based on reliability of ten acceptance criteria and the NCAP star rating using the response surface models. However, the verification study indicated that there is potential error in the response surface models and thus potential error in the reliability predictions. Several other limitations were identified during development and analysis of this model and warrant further investigation. Some of these limitations affect the performance models for each acceptance criteria:

1. Obtain LS-DYNA solutions to include the left rail stiffness as a random variable. The variation for this parameter did not yield complete LS-DYNA solutions and thus were not included in the performance models.
2. Improve the response surface models for acceptance criteria where warranted by the verification study.
3. Obtain model solutions for other important variables where a linear approximation was used.

These three items should be pursued prior to using these results for decision analysis.

In general, high-fidelity models are required to accurately predict vehicle performance for complex situations such as crash scenarios. These models are typically computationally intensive and multiple runs are required to predict reliability and for evaluating new designs. Practical application of this approach for performance models of this scale requires accurate response surface models of the performance measures. Several other development areas are identified to assist in future assessments:

1. Develop a meta-model capability to automate the response surface generation procedure used in this effort.
2. Include additional meta-models such as Kriging to enhance the predictive models by more accurately modeling the response of highly nonlinear applications encountered in impact analysis.
3. Develop an algorithm to automatically change the perturbation size when a response evaluation fails. This capability would assist in using fast probability integration methods such as the advanced mean value (AMV) method for stochastic crashworthiness analysis.

The results presented here show that probabilistic analysis can be used to effectively improve the design of complex systems. Improvements in automation of the analysis process and using combinations of efficient and robust probabilistic methods and approximate fast running performance models will lead to practical application of the developed approach for complex engineering design.

ACKNOWLEDGMENTS

This project was completed as a result of a grant from the 2000 DaimlerChrysler Challenge fund.

REFERENCES

1. LS-DYNA is a general-purpose transient dynamic finite element program. LS-DYNA is a registered trademark of Livermore Software Technology Corporation.

2. MADYMO combines multibody and finite element techniques for occupant safety analysis. MADYO is a registered trademark of TNO.

3. Hassan , J.E., Nusholtz,G. and Forrest, M. D., “A stochastic approach for occupant crash simulation”, SAE 2000-01-1597.

4. Southwest Research Institute, “Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion System Components,” Final Report NASA Contract NAS3-24389, NASA Lewis Research Center, Cleveland, Ohio, 1995.

5. Riha, D.S, Thacker, B.H., Enright, M.P., Huyse, L. and Fitch, S.H.K., "Recent Advances of the NESSUS Probabilistic Analysis Software for Engineering Applications," Proc. AIAA/ASME/ASCE/AHS/ASC 42nd Structures, Structural Dynamics, and Materials (SDM) Conf., AIAA-2002-1268, Denver, CO, 22-25 April 2002.

6. Riha, D.S and Manteufel, R.D., "Evaluation of Bounding Methods for Correlated Failure Modes," Proc. AIAA/ASME/ASCE/AHS/ASC 42nd Structures, Structural Dynamics, and Materials (SDM) Conf., AIAA 2001-1239, Seattle, WA, 16-19 April 2001.

7. Cruse, T. A. editor, 1997, Reliability-Based Mechanical Design, Marcel Dekker, Inc., New York, NY.

8. Torng, T.Y., Wu, Y.-T., and Millwater, H.R., "Structural System Reliability Calculation Using a Probabilistic Fault Tree Analysis Method," Proceedings 33rd SDM Conf., Part 2, pp. 603-613, April 1992.

9. Riha, D.S and Manteufel, R.D., “Convergence Studies of an Adaptive Importance Sampling Approach for System Reliability Analysis,” Proceedings of the Eighth Specialty Conference on Probabilistic Mechanics & Structural Reliability, Engineering Mechanics and Structural Divs, ASCE, PMC2000-165, University of Notre Dame, July 23-26, 2000.